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# VAPORS FOR HEAT ENGINES

INCLUDING

CONSIDERATIONS RELATING TO THE USE OF  
FLUIDS OTHER THAN STEAM FOR POWER  
GENERATION: A STUDY OF DESIRABLE VAC-  
UUM LIMITS IN SIMPLE CONDENSING EN-  
GINES: METHODS FOR COMPUTING EFFI-  
CIENCIES OF VAPOR CYCLES WITH LIMITED  
EXPANSION AND SUPERHEAT: A VOLUME-  
TEMPERATURE EQUATION FOR DRY STEAM:  
AND NEW TEMPERATURE-ENTROPY DIA-  
GRAMS FOR VARIOUS ENGINEERING VAPORS

BY

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WITH 21 TABLES AND 17 ILLUSTRATIONS



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# LIST OF ILLUSTRATIONS

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FIG.	PAGE
1. The Clausius Vapor Cycle . . . . .	4
2. Pressure-Temperature Relations of Engineering Vapors . . . . .	9
3. The Binary Vapor Principle . . . . .	26
3'. Turbine Characteristics . . . . .	29
4. Comparative Proportions of Power Plants, Complete Expansion Cycle . . . . .	32
5. Rankine Cycle for Dry Vapor . . . . .	35
6. Effect of Change in Back Pressure . . . . .	40
7. Comparative Proportions of Power Plants, Rankine Cycle . . . . .	56
8. Rankine Cycle with Superheat . . . . .	59
9. Temperature-Volume Curves for Dry Steam . . . . .	73
10. Temperature-entropy Chart for Alcohol . . . . .	44
11. Temperature-entropy Chart for Chloroform . . . . .	46
12. Temperature-entropy Chart for Acetone . . . . .	48
13. Temperature-entropy Chart for Carbon Chloride and Carbon Bisulphide . . . . .	51
14. Cyclic Efficiencies and Criterion . . . . .	53
15. Temperature-entropy Diagram for Ether . . . . .	37
16. Temperature-entropy Diagram for Ammonia . . . . .	71
17. Temperature-entropy Diagram for Carbon Dioxide . . . . .	74



# VAPORS FOR HEAT ENGINES

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## I

### General Considerations as to the Choice of a Working Fluid

WATER is the working substance in the great majority of external-combustion heat engines. It so far surpasses all other vapors in cheapness that it alone can be considered for use in a non-condensing cylinder. With condensing operation, the fluid may be mostly saved, to be used over and over again in a closed cycle; and if the loss by leakage is not too great, some other vapor may replace steam.

To get a rough idea of the amount of leakage permissible, suppose a pound of coal, containing 14,000 B.T.U., to generate steam at 70 per cent efficiency, so that

$$14,000 \times 0.70 = 9800 \text{ B.T.U.},$$

are contained in the steam. Let this heat perform work in a steam engine at 10 per cent efficiency,

$$9800 \times 0.10 = 980 \text{ B.T.U.},$$

being converted into useful work per pound of coal burned. Suppose also that some other fluid were  $\frac{1}{10}$  more efficient

than steam, i.e., that it could drive a heat engine at 11 per cent efficiency. In obtaining 980 B.T.U. of useful work we should then consume

$$980 \div 0.11 = 8909 \text{ B.T.U.}$$

of heat in the vapor of the assumed fluid. If this fluid could be generated at the same boiler efficiency as steam, the heat necessary in the coal would be

$$8909 \div 0.70 = 12,727 \text{ B.T.U.,}$$

a saving of  $14,000 - 12,727 = 1273$  B.T.U. or 9.1 per cent, in fuel cost, which might offset the expense due to leakage of fluid in operation.

Suppose this new fluid to cost, pound for pound, the same as coal: leakage amounting to 9.1 per cent would be permissible; if it costs twice as much as coal, the allowable limit of leakage would be 4.55 per cent; if it costs ten times as much as coal, the leakage limit is  $\frac{9.1}{10}$  of one per cent, and so on. There are volatile vapors which might be used in heat engines, costing not more than ten times as much as coal; and there are vapors permitting, under certain conditions, of an efficiency exceeding by  $\frac{1}{10}$  that attainable with steam. With a leakage loss below 1 per cent, an investigation of the possibilities in applying these vapors to power production should be not without interest.

Engines have actually been built using ether, sulphur dioxide, gasoline, alcohol and ammonia, among other vapors besides steam. Ammonia, not steam, is the fluid commonly used in the cylinders of refrigerating compressors; steam would of course not answer; but the expense due to leakage of ammonia is not ordinarily a matter of vital importance.

## II

### Data for the Analysis

IN what follows, it will be assumed that the Carnot formula,

$$\frac{T-t}{T}, \quad . . . . . (A)$$

is recognized as an expression in terms of the absolute temperatures for the ideal limiting efficiency of any heat engine whatever, working between the temperatures specified.

For a vapor engine, however, there is an equally definite and lower limit of efficiency. Some acquaintance with the temperature-entropy diagram must now be assumed. In Fig. 1, ordinates are absolute temperatures, abscissas are entropies, horizontal lines are isothermals (and also, for saturated vapors, lines of constant pressure), vertical lines are adiabatics. The area under any line, down to the *ON* axis, represents the heat absorbed or emitted in working the substance along the corresponding path. The ideal cycle for a vapor initially dry is *abcd*, *ab* being the path of constant pressure and nearly constant specific heat followed in heating the liquid, *bc* the path of vaporization, *cd* that of adiabatic expansion, and *da* that of condensation at constant pressure. The cycle *lbcd* is that of Carnot, bounded by isothermals and adiabatics alone. It is an impracticable cycle for a vapor.

The efficiency of the cycle  $abcd$  is,

$$\frac{\text{Heat converted into work}}{\text{Gross amount of heat expended}} = \frac{abcd}{cahcf}$$

$$= \frac{cabk + kbcf - eadf}{cabk + kbcf} \quad (B)$$

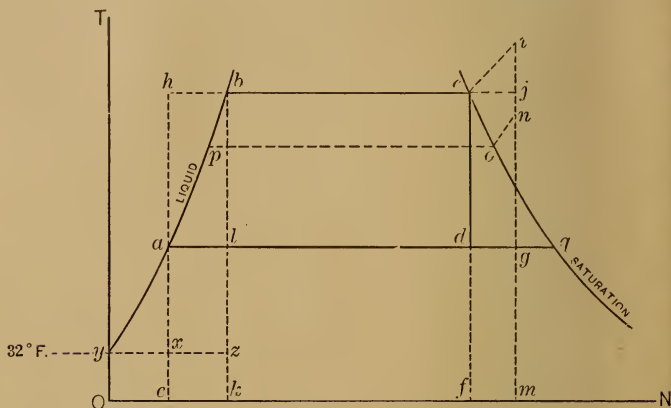


FIG. 1.—The Clausius Vapor Cycle

If the upper and lower absolute temperature limits of the cycle be  $T$  and  $t$  respectively,  $L$  and  $l$  being the corresponding heats of vaporization, then

$$bc = \frac{L}{T}, \quad da = \frac{l}{t};$$

and if the specific heat of the liquid be constant and equal to  $c$ ,

$$al = \int \frac{dH}{T} = \int \frac{cdT}{T} = c \int_t^T \frac{dT}{T} = c \log_e \frac{T}{t},$$

the  $H$  and  $T$  in the first integral denoting heat and temperature, respectively, in general. If  $h_b$  and  $h_a$  denote the

respective heats of the liquid corresponding with the absolute temperatures  $T$  and  $t$ , we have as a definite expression for the efficiency <sup>1</sup> of the cycle  $abcd$ ,

$$\frac{h_b - h_a + L - \frac{ad}{aq} \cdot l}{h_b - h_a + L},$$

$$= 1 - t \cdot \frac{c \log_e \frac{T}{t} + \frac{L}{T}}{h_b - h_a + L} \cdot \dots \dots \dots (C)$$

Eq. (C) is, however, inapplicable for the purpose in hand, because  $c$  is in general quite variable. We may use successive values of  $c$  for computing changes of entropy for short temperature ranges and thus obtain a close approximation to the change for any finite range.

The values of  $c$  over the short temperature ranges chosen in the exemplifying table on page 6, are, of course, obtained by dividing the differences of "heats of liquid" by those temperature ranges. The expression  $\frac{T}{t}$  denotes the quotient of absolute temperatures expressing the range.

If now we sum up the figures in the last column, we shall have a series of figures representing the entropies of liquid (abscissas of the path  $y'b$  in Fig. 1) at various temperatures, these entropies being tabulated above 32° F. as an arbitrary

<sup>1</sup> This statement of efficiency has been preferred by the writer, although some authorities compute efficiencies on the basis of heat absorbed above 32°, making the denominator in Eq. (C) simply  $H_c$  (total heat in dry steam) = the area  $oybcf$ , Fig. 1. But even in bad practice water is fed to the boiler at a higher temperature than 32°; so that it seems reasonable, in establishing ideal standards, to assume it to be delivered thereto at the temperature at which it is rejected by the engine.

TABLE I

## COMPUTATION OF ENTROPY OF LIQUID OF ALCOHOL

Temperature, ° F.	Heat of the Liquid	Differences.		Corre- sponding Value of $c$	$\frac{T}{t}$	$c \log_e \frac{T}{t}$
		Tempera- ture	Heat of Liquid			
32	0	—	—	—	—	—
50	10.06	18	10.06	0.559	$\frac{510}{492} = 1.036$	0.0198
68	20.56	18	10.50	0.583	$\frac{528}{510} = 1.035$	0.0204
86	31.48	18	10.92	0.607	$\frac{546}{528} = 1.033$	0.0198
104	42.68	18	11.20	0.622	$\frac{564}{546} = 1.032$	0.0200
122	54.38	18	11.70	0.650	$\frac{582}{564} = 1.031$	0.0206
140	67.27	18	12.89	0.716	$\frac{600}{582} = 1.031$	0.0227
158	80.24	18	12.97	0.721	$\frac{618}{600} = 1.030$	0.0214
176	93.80	18	13.56	0.753	$\frac{636}{618} = 1.029$	0.0211
194	107.95	18	14.15	0.786	$\frac{654}{636} = 1.028$	0.0217
212	122.72	18	14.77	0.821	$\frac{672}{654} = 1.027$	0.0223
230	138.13	18	15.41	0.856	$\frac{690}{672} = 1.027$	0.0232
248	154.21	18	16.08	0.894	$\frac{708}{690} = 1.026$	0.0230
266	170.96	18	16.75	0.931	$\frac{726}{708} = 1.025$	0.0225
284	188.46	18	17.50	0.972	$\frac{744}{726} = 1.024$	0.0230
302	206.68	18	18.22	1.012	$\frac{762}{744} = 1.023$	0.0235



starting point. The distance  $al$  on the diagram may then be written,

$$al = yz - yx = n_b - n_a,$$

where  $n_b$  and  $n_a$  denote, respectively, these tabulated entropies of the liquid for the two points specified. The symbol  $n_w$  will be employed for entropy of liquid in general, measured at over 32° F.

Again, widths like  $bc$ , from the liquid to the saturation curve, are always equal to the quotient of latent heat of vaporization by absolute temperature; that is, to

$$\frac{L}{T} \quad \text{or} \quad \frac{l}{t}.$$

These quantities may also be tabulated for various temperatures, the general symbol being  $n_e$ ; or, referring to Fig. 1,  $n_{bc}$ ,  $n_{aa}$ , etc. Finally, by adding the values of  $n_w$  and  $n_e$ , for any temperature, we have that of  $n_s$ , the total entropy of the dry vapor at the same temperature.

These three properties<sup>1</sup> have been tabulated for all of the vapors to be considered (except steam) in Table XXI.

We may now write Eq. (B) in the form,

$$\text{Efficiency} = \frac{h_b - h_a + L - t(n_b - n_a + n_{bc})}{h_b - h_a + L}, \quad \dots \quad (D)$$

which is the exact expression for the cycle with complete adiabatic expansion.

<sup>1</sup> The vapor properties employed in this discussion have been taken from the appendices to Vol. II of Zeuner's *Technical Thermodynamics*, Klein Edition (D. Van Nostrand Co.). The tables of entropies were, however, compiled by the writer from data given by Zeuner, especially for the present work.

**The Pressure-temperature Relation.** This must be clearly understood: that a fluid boils at a definite temperature for every pressure to which it may be subjected; the greater the pressure, the higher is this temperature; a vapor cannot exist, as such, at a temperature below that which thus "corresponds" with its pressure; but, by *superheating*, it may be brought to any *higher* temperature desired. The lower the pressure at which a non-superheated vapor is formed, the greater is the space which it occupies; and (an important fact in the subsequent discussion) this even holds in a rough, approximate way for vapors generally, so that if for any boiling-point we should tabulate the corresponding pressures of a number of vapors, and afterward the spaces occupied by unit weight (the *specific volumes*) of the same vapors, we should find that they ranked, in order of pressures, somewhat inversely as they ranked in order of specific volumes.

In Fig. 2, we have plotted the pressure-temperature curves of various vapors from the figures given in Table II. The curve for steam occupies the extreme right-hand position; i.e., its pressure, at a given temperature, is less than that of any other vapor considered.

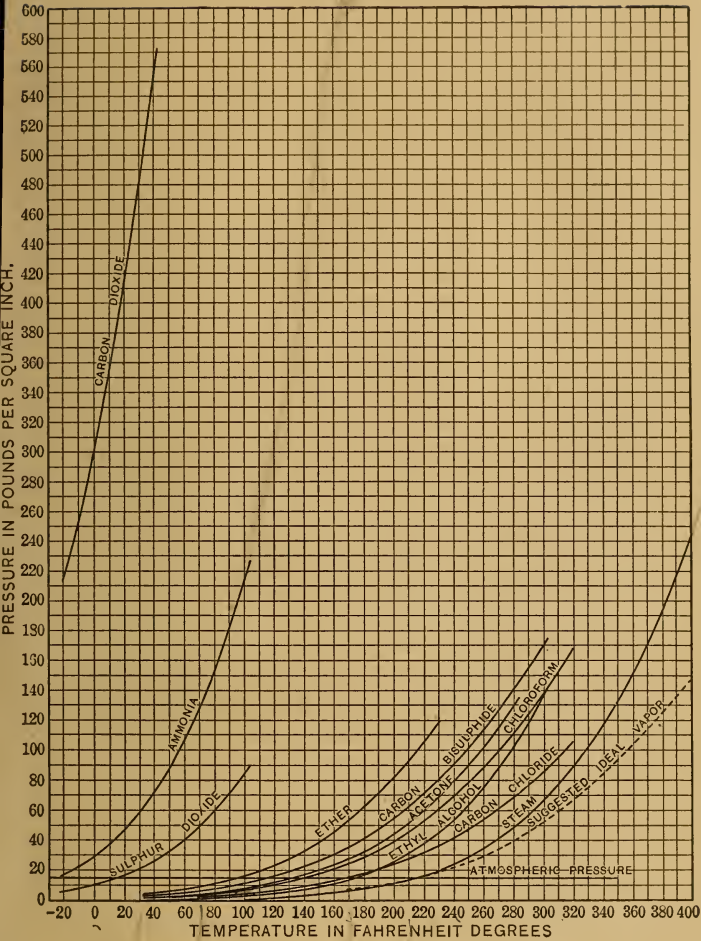


FIG. 2.—Pressure-Temperature Relations of Engineering Vapors

### III

#### The Limit of Efficiency with Steam

THE maximum pressure at which steam is commonly worked on a commercial scale is 250 lbs., corresponding to a temperature of, very nearly, 400° F. With superheat, temperatures up to 600°, or even higher, are employed. These limits of temperature may be easily accounted for on mechanical and commercial grounds. They are not necessarily permanent.

There are equally definite (and far more permanent) limits of lower temperature. A vapor cannot exist at a temperature lower than that corresponding with its pressure. If an engine exhausts into the atmosphere, the temperature of heat rejection cannot be less than that corresponding with a pressure of 14.696 lbs. per square inch—212° F. If it exhausts into a vacuum of 28 inches of mercury, the corresponding temperature is 100° F. Lower than these temperatures we cannot go. The best average vacuum that can be commercially maintained is not over 28 ins. The pressure of steam is then only 0.946 lb. per square inch. It has been brought to its condition of greatest attenuation outside the laboratory. By applying Eq. (A), taking the Fahrenheit zero as 460° above the zero absolute,



we have the following as the ideal limits of efficiency in steam engines according to present practice:

$$\text{Non-condensing, saturated steam, } \frac{400-212}{400+460} = 0.22;$$

$$\text{Condensing, saturated steam, } \frac{400-100}{400+460} = 0.35;$$

$$\text{Condensing, superheated steam, } \frac{600-100}{600+460} = 0.47.$$

The best *actual* (cylinder) thermal efficiency ever recorded for a vapor engine was about 0.25.

## IV

### The Line of Attack

SUPPOSE we assume a pressure of 250 lbs. or a temperature of  $600^{\circ}$  (with superheat) to determine the upper limit of the cycle, and a vacuum of 28 inches to fix the lower limit. The use of a vapor other than steam might then, apparently, be justified on one of three grounds:

1. A higher temperature might be attained at 250 lbs. pressure, without superheat.

2. A temperature of  $600^{\circ}$  might be attained by superheating, with an efficiency higher than is possible from steam at that temperature.

3. A lower temperature might be attained at 28 ins. of vacuum.

If, as in Fig. 2, we plot curves with temperatures as abscissas and corresponding pressures as ordinates, conditions 1 and 3 taken together require a vapor giving a curve which *crosses that for steam*. At a very low pressure, we wish the boiling-point to be lower than that of steam; and at higher pressures, its boiling-point should be the greater. No such vapor is known to the writer. It is not impossible that one may exist, for similar crossings of the pressure-temperature curves occur with other pairs of fluids. For example, the carbon chloride and ethyl alcohol curves, in Fig. 2, show a crossing point near  $190^{\circ}$  F.; our hypothetical vapor should give a curve related to that of steam in much



the same way as the carbon chloride curve is related to that of alcohol. Its properties would have to be somewhat as indicated by the dotted line.

Condition 1 is easy to meet. There are many known vapors giving curves lying wholly to the *right* of the steam curve in Fig. 2; but these vapors have the general disadvantage of giving a higher temperature than steam at 28 ins. of vacuum, so that condition 3 is violated.

Condition 3 would lead to a lower temperature of heat-rejection, and thus increase the potential efficiency of the cycle; but in any case this temperature cannot be below that of the *average available supply of cooling water*; or, say, in our latitude, about 60° F. The limit of efficiency with superheat, for *any vapor*, would then be, from Eq. (A),

$$\frac{600-60}{600+460}=0.51,$$

an increase of about 8 per cent over the present limit with steam.

If we disregard the possible existence of such a vapor as is designated by the dotted line of Fig. 2, and for the present restrict the discussion to saturated (non-superheated) vapors, we must obviously dwell upon Condition 3. The vapor to be preferred is one which boils at about 60° F. (any lower temperature is needless, on account of the cooling water limit, and likely to lead to excessive maximum pressures) at an absolute pressure of from 1 to 4 lbs. per square inch. Carbon bisulphide, chloroform, acetone and carbon chloride (with possibly alcohol) are the only fluids to be considered (see Appendix I). The last (carbon chloride) requires the best vacuum—a moderate one, however—but is most desirable from the standpoint of maximum pressure, as indicated on page 15.



TABLE III

## MAXIMUM PRESSURES WITH VARIOUS VAPORS

Vapor	Pressure in Lbs. per Sq.in., at 302° F.
Alcohol.....	142
Chloroform.....	140
Acetone.....	163*
Carbon bisulphide.....	176
Carbon chloride.....	88
Steam.....	69

\* Extrapolated.

The pressure with carbon chloride is only 28 per cent greater than that with steam. The indication is that at higher temperatures the excess percentage will be less, the two curves in Fig. 2 perhaps crossing near 400° F., our assumed upper limit with saturated steam. It may then be that the "suggested ideal vapor" of Fig. 2 really exists, as carbon chloride, the curve for which resembles the dotted curve shown, excepting that it occupies a position further toward the left. In such case, the objection to the use of carbon chloride as a substitute for steam, with an accompanying 8 per cent increase in potential efficiency, is the probable expense due to leakage.

## V

### Efficiencies of Dry Vapors in the Complete Expansion Cycle

CONSIDERING now the diagram  $abcd$  of Fig. 1, to which Eqs. (B), (C) and (D) apply, we may examine the efficiencies shown by these equations as representing more nearly the limits of practice. It is first proposed to establish a criterion for estimating relative efficiencies in advance.

The cycle  $abcd$  is less efficient than the Carnot cycle  $ahcd$  drawn through the same extreme limits. The work areas are: Carnot,  $ahcd$ : Clausius<sup>1</sup>,  $abcd$ ; and the amounts of heat chargeable are; Carnot,  $ehcf$ ; Clausius,  $eabcf$ . The excess of heat chargeable in the case of the Carnot cycle

<sup>1</sup>It is a current, but (the writer believes) unjustifiable, habit to refer to the complete expansion cycle  $abcd$ , Fig. 1, as Rankine's. It does not appear that Rankine ever described such a cycle, although Clausius did, in his Fifth Memoir on the Application of the Mechanical Theory of Heat to the Steam Engine. Rankine shows the adiabatic expansion cycle with terminal drop (incomplete expansion) in *The Steam Engine*, 1897 Ed., Art. 278; and this is the cycle which should properly be associated with his name. The Clausius (complete expansion) cycle is perfectly definite for a given dry vapor. The temperature limits fully determine the efficiency, just as in the Carnot cycle. It is the ideal cycle of a vapor engine. The terminal drop cycle, on the other hand, is indefinite and establishes no standard for comparison with results obtained in actual engines. Any number of such cycles, of various degrees of efficiency, is possible between two given temperature limits.

is  $ahb$ ; but the excess of work obtained is also  $ahb$ , so that this work is obtained at 100 per cent efficiency. Whatever makes the Clausius cycle more nearly like that of Carnot increases the efficiency of the former. The ideal work area should be rectangular, not trapezoidal. Departure of the area  $abcd$  from rectangular form is due wholly to the slope of the line  $ab$ . This slope varies with the specific heat of the liquid, since the entropy or abscissa of the path  $ab$ , ( $al$ ), has been shown to be equal to  $c \log_e \frac{t_b}{t_a}$ , where  $c$  is that specific heat. When  $c=0$ ,  $ab$  is vertical and  $abcd$  is a rectangle.

Further, slope of the line  $ab$  becomes less important in producing deviation of the area  $abcd$  from rectangular form when the width  $bc$  is relatively great. This width is  $\frac{L}{T}$ , the quotient of the latent heat of vaporization by the absolute temperature. At any temperature, then, it is desirable that the latent heat should have a high value. Considering both factors, *the most efficient fluid is likely to be that for which there is obtained at a given temperature, the maximum value of*

$$\frac{L}{c} = \frac{\text{Latent heat of vaporization}}{\text{Specific heat of the liquid}}$$

The following table applies this (not new) principle to Fig. 1.

TABLE IV  
 PRELIMINARY ESTIMATE OF VAPOR EFFICIENCIES BETWEEN 302° AND 68° F.

VAPOR	$h_b$	$h_a$	$\frac{h_b - h_a}{302 - 68} = c$ (mean)	$L_{bc}$	$l_{aq}$	$\frac{L_{bc} + l_{aq}}{2}$ (mean)	$L + c$ (mean)	Order of Merit
Alcohol.....	206.68	20.56	$\frac{186.12}{234} = 0.800$	306.86	433.04	369.95	462	3
Chloroform.....	64.78	8.41	$\frac{56.37}{234} = 0.241$	92.94	117.14	105.04	436	4
Acetone.....	152.76*	18.52	$\frac{134.24}{234} = 0.575$	178.11*	247.20	212.66	370	6
Carbon bisulphide...	66.82	8.53	$\frac{58.29}{234} = 0.250$	117.90	158.44	138.17	552	2
Carbon chloride.....	57.11	7.18	$\frac{49.93}{234} = 0.214$	69.02	91.57	80.30	375	5
Steam.....	271.60	36.07	$\frac{235.53}{234} = 1.010$	908.00	1053.40	980.70	970	1

\* Extrapolated.

Steam appears likely to give the most efficient cycle, carbon chloride nearly the least efficient. If we now apply Eq. (D), taking additional values from the entropy tables, page 78, we obtain:

## DEFINITIVE VAPOR EFFICIENCIES

Alcohol,

$$\frac{186.12 + 306.86 - 528(0.3250 - 0.0402 + 0.403)}{492.98} = 0.264;$$

Chloroform,

$$\frac{56.37 + 92.94 - 528(0.1077 - 0.0172 + 0.1219)}{149.31} = 0.249;$$

Acetone,

$$\frac{134.24^* + 178.11^* - 528(0.2465^* - 0.0368 + 0.236^*)}{312.35} = 0.246;$$

Carbon bisulphide,

$$\frac{58.29 + 117.90 - 528(0.1099 - 0.0172 + 0.154)}{176.19} = 0.260;$$

Carbon chloride,

$$\frac{49.93 + 69.02 - 528(0.0953 - 0.0145 + 0.0905)}{118.95} = 0.238;$$

Steam,

$$\frac{235.53 + 908.00 - 528(0.4398 - 0.0707 + 1.1921)}{1143.53} = 0.279;$$

the order of efficiencies being substantially as predicted. The efficiency of the Carnot cycle between the same temperature limits is

$$\frac{302 - 68}{302 + 460} = 0.306.$$

\* Extrapolated.

But while four of the six vapors could in practice be worked down to a temperature of 68° F. without difficulty, this would be impossible with steam or with alcohol. Steam, for example, would at this temperature exert a pressure of only 0.34 lb. per square inch, equivalent to a vacuum of 29.2 ins. With a more practicable lower temperature limit—say 110° F.—the efficiency of the Clausius cycle, using steam, would be (from Eq. (D)) only

$$\frac{271.60 - 77.94 + 908.00 - 570(0.4398 - 0.1471 + 1.1921)}{271.60 - 77.94 + 908.00} = 0.231;$$

slightly lower than that obtainable with the better temperature range possible with the other vapors. A reduction in lower temperature limit is thus shown likely to be profitable (though only slightly so) when the matter is viewed under conditions more closely corresponding to practice than those of the Carnot cycle.

It appears, then, that an engine using the vapors of chloroform, acetone, carbon bisulphide or carbon chloride would, with a less perfect vacuum than is now common in steam plants, permit of an efficiency somewhat exceeding that attainable with steam. Expansion is assumed to be complete, and (in some cases at least) the maximum pressure would be increased.

## VI

### Superheat

THIS conclusion is unsatisfactory in that some excess of initial pressure is involved. It is true that with carbon chloride the excess is not great, and might at the upper limit of  $400^{\circ}$  become *nil*; but with this vapor the gain in efficiency is also small and might disappear if  $400^{\circ}$  F. were fixed as the upper temperature.

We might avoid excessive pressures by superheating, while at the same time increasing efficiency; but steam can be superheated as well as the other vapors. We have assumed that it is virtually a pressure condition, rather than a temperature condition, which establishes the *lower* limit of our cycle; the vacuum necessary must not exceed about 28 ins. of mercury. Let us now accept a pressure condition as also establishing the upper limit and examine a superheated cycle in which a maximum temperature of  $600^{\circ}$  is attained at a pressure not exceeding 100 lbs. per square inch.

Is there in this case any criterion from which we may hazard an advance guess, as with the saturated vapor cycles, regarding the probable order of efficiencies? The area *abcig*, Fig. 1, represents the operation to be considered. Comparing it with the former cycle *abcd*, we find the added work area, *dcig*, to consist of the two parts *dcjg* and *cij*. The increased expenditure of heat may similarly be divided into the two parts; *fejg*, giving the work area *dcjg* (a Carnot

cycle), and  $cij$ , giving a work area equivalent to itself. The first work area is necessarily obtained at slightly greater efficiency than the original Clausius cycle  $abcd$  between the same temperature limits. The latter work area is equal to the heat which it costs. It is gained at 100 per cent efficiency, and is the potent factor in making the cycle  $abcig$  more efficient than  $abcd$ .

The extent to which superheating will increase cyclic efficiency is thus closely related to the ratio of the areas  $cij$ ,  $abcd$ . The former area will be large when  $T_i - T_c$  is large and when the width  $cj$  is large. The former condition is approached as  $T_c$  decreases, for  $T_i$  has been fixed at  $600^\circ$  F. We have also established the pressure at  $c$  as 100 lbs. We wish, then, for a vapor in which the saturation temperature at 100 lbs. pressure is relatively low; that is, a vapor lying to the left of the steam curve in Fig. 2.

Again, the width  $cj$  is directly proportional to the specific heat of the superheated vapor. The slope of the constant pressure path of superheat,  $ci$ , is related to this constant, just as the slope of the liquid line,  $ab$ , is related to the specific heat of the liquid. Considering both conditions, then, the most desirable vapor will be that in which

- (a) The specific heat during superheating is large, and
- (b) The temperature at 100 lbs. pressure is small.

Superheated steam has, undoubtedly, the highest specific heat of any of the vapors under discussion. That of alcohol (0.4534) approaches it, while that of chloroform, for example, is only 0.1567. Whether the better pressure-temperature relations of these two vapors may offset their less desirable specific heat values can be determined only by computing the efficiencies in detail. The advance criterion of efficiency is in this type of cycle indefinite; but we may at least presume that alcohol will give a more favorable result than chloroform, on account of its much higher specific heat.



We will then examine the cycle in which  $T_i = 600^\circ \text{ F.}$ , the maximum pressure is 100 lbs., and the lower pressure is not less than 1 lb. For steam, at 1 lb. absolute pressure, the lower temperature limit will be  $101.83^\circ$ ; for alcohol, it will be  $72^\circ$ ; while for chloroform, since it would be only  $28^\circ$ , we will regard the cooling water as establishing a lower limit at  $60^\circ \text{ F.}$  The following thermal properties are tabular:

TABLE V

VAPOR PROPERTIES FOR CYCLES WITH SUPERHEAT

Vapor.	$T_c$	$h_b$	$t_a$	$h_a$	$L_{bc}$	$n_c$	$k^*$	$n_a$
Steam . . . .	327.8	298.3	101.83	69.8	888.0	1.602	0.52	0.1327
Alcohol . . .	277.0	181.8	72.00	23.0	320.0	0.728	0.4534	0.0446
Chloroform	270.3	57.0	60.00	6.54	96.37	0.229	0.1567	0.0134

\* Specific heat of the superheated vapor.

The expression for efficiency is, if the expansion line crosses the saturation curve,

$$\frac{abcig}{eabcim} = \frac{abke + kbcf + fcim - agme}{abke + kbcf + fcim}$$

$$= 1 - \frac{(n_i - n_a)t_a}{h_b - h_a + L_{bc} + k(T_i - T_c)} \quad (E)$$

Where  $n$  is the symbol for entropy above  $32^\circ \text{ F.}$  In order to find  $n_i$ , we write,

$$n_i - n_c = k \log_e \frac{T_i}{T_c}, \quad \dots \quad (F)$$

$n_c$  being tabular and  $T_i$  always  $600 + 460 = 1060$ . Then,

For steam, 
$$n_i = 1.602 + \left( 0.52 \times 2.3 \log \frac{1060}{787.8} \right) = 1.7558;$$

For alcohol,  $n_i = 0.728 + \left(0.4534 \times 2.3 \log \frac{1060}{737.0}\right) = 0.8925$ ;

and

For chloroform,  $n_i = 0.229 + \left(0.1567 \times 2.3 \log \frac{1060}{730.3}\right) = 0.2874$ .

But in the general case the vapor may remain superheated at the end of expansion, giving such a cycle as *pbcin*<sub>o</sub>, Fig. 1. To determine whether this is the case for our conditions, we have only to compare values of  $n_i$  and  $n_o$ , the latter being the total entropy of the dry vapor at the lower temperature, and having the following values: for steam, 1.9754; for alcohol, 0.857; and for chloroform, 0.2416. Since  $n_o$  exceeds  $n_i$  for steam, the cycle is like *abcig*, and Eq. (E) is applicable, yielding,

$$\text{For steam, } 1 - \frac{(1.7558 - 0.1327)561.83}{228.5 + 888.0 + 0.52(272.2)} = 0.272.$$

But for alcohol and chloroform another equation must be found, applicable to such a cycle as *pbcin*<sub>o</sub>. This equation is

$$\begin{aligned} \text{Efficiency} &= \frac{pbcin_o}{h_b - h_p + L_{bc} + k(T_i - T_c)} \\ &= 1 - \frac{L_{po} + k(T_n - T_o)}{h_b - h_p + L_{bc} + k(T_i - T_c)}, \quad \dots \quad (G) \end{aligned}$$

in which  $T_n$  is to be found from the relation

$$n_i - n_p = n_{po} + n_{on} = n_{po} + k \log_e \frac{T_n}{T_o} \quad \dots \quad (H)$$

The values of  $L_{po}$  are respectively 433.01 and 117.91; those of  $n_{po}$  are 0.814 and 0.2278. Applying Eq. (H),

For alcohol,  $0.8925 - 0.0446 = 0.814 + \left(0.4534 \times 2.3 \log \frac{T_n}{532}\right)$ ,

and

$$T_n = 571^\circ \text{ absolute or } 111^\circ \text{ F.};$$

while for chloroform,

$$0.2874 - 0.0134 = 0.2278 + \left( 0.1567 \times 2.3 \log \frac{T_n}{520} \right),$$

and

$$T_n = 706^\circ \text{ absolute or } 246^\circ \text{ F.}$$

The corresponding efficiencies, from Eq. (G), are

$$1 - \frac{433.01 + 0.4534(111 - 72)}{478.8 + (0.4534 \times 323)} = 0.278 \quad \text{for alcohol,}$$

and

$$1 - \frac{117.91 + 0.1567(246 - 60)}{146.83 + (0.1567 \times 329.7)} = 0.259 \quad \text{for chloroform,}$$

confirming the prediction made, in spite of the greater temperature range with chloroform. Alcohol and steam are about equally efficient, while chloroform is decidedly less desirable under these superheated conditions. The values of  $k$  taken are somewhat uncertain, and this property is too variable to warrant our drawing any closer conclusions; but it seems safe to say that there is no inherent advantage on the part of either of the proposed vapors in a complete expansion condensing engine using superheated steam; the three efficiencies seem to have no relation to the condenser temperature. We cannot by superheating, consequently, evade the high initial pressures to which exception has been taken, without at the same time losing the efficiency advantage shown under certain circumstances to be possible.

## VII

### The Binary Vapor Principle

HIGH initial pressure may, however, be eliminated by the vapor engine of Du Tremblay, in which steam, discharged from a cylinder at, say 110° F., may be condensed by the

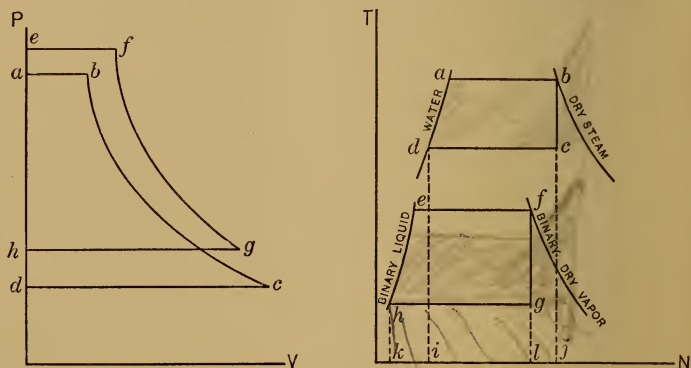


FIG. 3.—The Binary Vapor Principle

*abcd* = primary

*efgh* = binary

Ideally, *idcj* = *khefl*

circulation of a more volatile fluid in the condenser coils. This second fluid is thus vaporized and may be used to perform work in a second cylinder. We may thereby work down to the cooling water limit of temperature—about 60° F.—and so obtain the slight increase in efficiency that our calculations have shown to be possible, without any increase in maximum pressure. Commercially, this

gain is insufficient to offset the added complications. The principle has been applied, intermittently, in actual engines for at least sixty years, with the expected economical thermal result, if not with commercial success. Fig. 3 shows the combined ideal indicator and entropy diagrams. The initial and back pressures on the two cylinders will usually differ, though not always in the way here indicated. There might be a mechanical advantage in having them equal.

## VIII

### Application to the Turbine

THE Clausius cycle analyzed is that of the steam turbine rather than that of the reciprocating engine; in which latter, cylinder condensation makes anything like complete expansion undesirable. The nozzle velocities obtained from a frictionless adiabatic flow, adopting the usual approximate formula,  $V = 224\sqrt{H}$ , where  $H$  is the cyclic area in B.T.U., may be computed as follows: these cyclic areas are the numerators of the efficiency expressions given in sections V and VI, so that if the efficiencies be each multiplied by their respective denominators we have at once the required numerators.

TABLE VI  
CYCLIC AREAS AND NOZZLE VELOCITIES

VAPOR	$H$	$V$
(a) 302° TO 68°, VAPORS INITIALLY DRY		
Alcohol.....	0.264 × 492.98 = 130.0	2550
Chloroform.....	0.249 × 149.31 = 37.1	1360
Acetone.....	0.246 × 312.35 = 77.0	1960
Carbon bisulphide..	0.260 × 176.19 = 45.9	1520
Carbon chloride....	0.238 × 118.95 = 28.3	1190
Steam.....	0.279 × 1143.53 = 319.0	4000
(b) INITIALLY DRY VAPOR, 302° TO 110°		
Steam.....	0.231 × 1101.66 = 255.0	3570
(c) VAPORS WITH SUPERHEAT AT 600° F.		
Steam.....	0.272 × 1257.5 = 342	4150
Alcohol.....	0.278 × 625.0 = 174	2960
Chloroform.....	0.259 × 198.48 = 51.3	1600

The variation in velocities is notable. These velocities are of course proportional to the square roots of the quan-

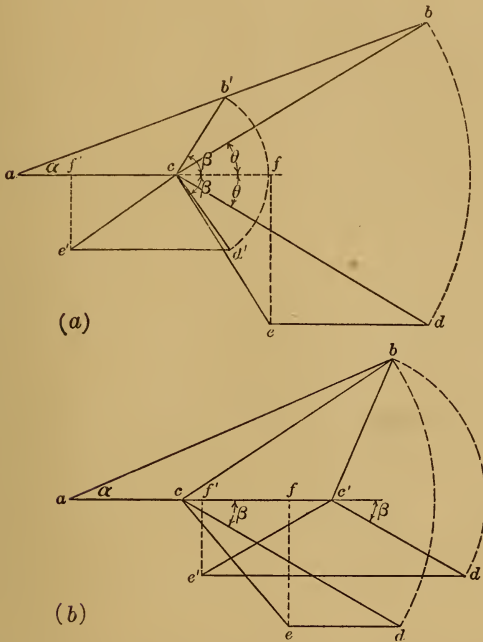


FIG. 3'.—Turbine Characteristics with Frictionless Buckets

	(a)	(b)
Jet velocity. . . . .	$ab, ab'$	$ab$
Peripheral velocity. . . . .	$ac, ed$	$ac, ac', ed, e'd'$
Absolute exit velocity. . . . .	$ec, e'c$	$e'c', ec$
Rotative components of } absolute exit velocities }	$cf, cf'$ (negative)	$cf, c'f'$ (negative)

tities of heat converted into work in the various cycles considered; and in the actual working out of a turbine design, the question of absolute emerging velocity is fundamentally related both to mechanical limitations and to the obtained

efficiency. Our efficiency equations have been applicable to ideal conditions only. The velocity of flow will be an important factor in determining how nearly the actual turbine will approach the ideal efficiency.

To consider this subject in all of its bearings would require a somewhat extended discussion. We may briefly point out three facts:

(1) With a given nozzle angle and peripheral speed, and with buckets of usual form a relatively low nozzle velocity is apt to lead to a retarding reaction at exit. (See Fig. 3', *a*.)

(2) With a given nozzle angle and nozzle velocity, positive exit reactions are associated with the lower peripheral speeds. (See Fig. 3', *b*.)

(3) With a given nozzle angle the peripheral speeds of impulse turbines using various vapors will with usual bucket angles vary about as the nozzle velocities of those vapors.

An efficient velocity turbine would therefore be possible at low peripheral speeds, with these special vapors, without excessive compounding into pressure stages.



## IX

### Some Commercial Considerations

**Boiler Capacity.** The argument is sometimes advanced, in connection with alcohol vapor launch engines, that the low value of the latent heat of vaporization of this fluid is an advantage in that less time and less boiler surface are required to "get up steam."

The size or capacity of a steam boiler is measured by its heating surface. Under the conditions which normally exist a heat transmission of about 33,000 B.T.U., per square foot of surface per hour, is considered reasonable. Very nearly the same conditions hold, regardless of the particular fluid contained in the boiler. With boilers of a given type, the quantity (volume) of fluid contained will bear a fairly constant ratio to the heating surface and therefore to the heat transmission.

In a power plant, the efficiency of the engine determines the quantity of heat to be supplied by the vapor leaving the boiler, per horse-power-hour. This efficiency, therefore, determines also the heating surface of the boiler, and, from the conclusion already reached, it determines the volume of liquid in the boiler.

The "time to get up steam" for a given volume of liquid in the boiler will depend also upon the *specific* volume of that liquid. Finally, therefore, boiler capacities necessary with various fluids may be expected to vary inversely as the cyclic efficiencies; the times consumed in starting

up the boilers will vary inversely as the products of efficiency by specific volume of liquid. The data for comparison are given in Table VII, volumes being taken at 212° F. The "quick steaming" boiler will, however, lack steadiness, and the comparison really means very little, for quickness

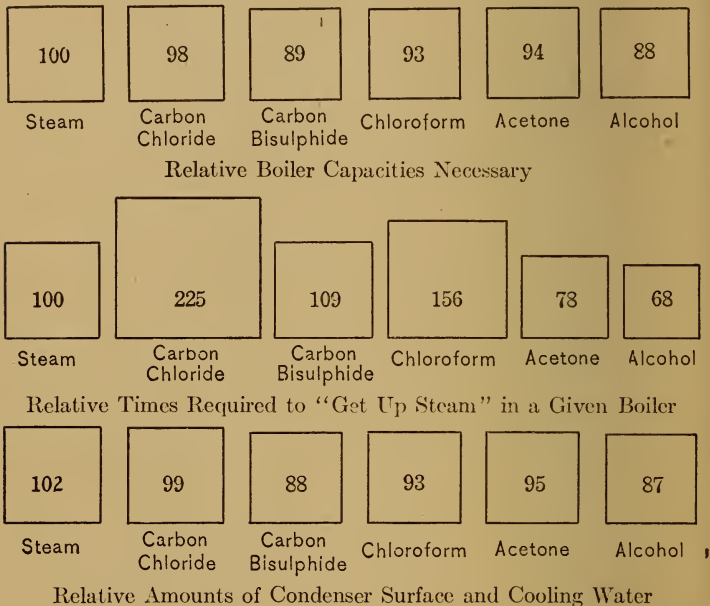


FIG. 4.—Comparative Proportions of Power Plants Using Various Fluids in the Complete Expansion Cycle, All Initially Dry Vapors and All Developing the Same Horse-power

of steaming might be attained in any case by using a type of boiler having a low ratio of liquid contents to heating surface.

**Cooling Water.** In Fig. 1, the area  $abcd$  represents work done and the area  $eadf$  represents heat which must be removed by the condenser. The ratio of the latter area to the for-

TABLE VII  
BOILER CAPACITY AND STEAMING RATE  
CLAUSIUS CYCLES WITH VAPOR INITIALLY DRY

Vapor	Efficiency	Relative Boiler Capacity Necessary	Volume of Liquid	Volume $\times$ Efficiency	Relative Time to "Get up Steam"
Alcohol . . . . .	0.264	88	0.0208	0.00549	68
Chloroform . . . . .	0.249	93	0.0096	0.00238	156
Acetone . . . . .	0.246	94	0.0192	0.00472	78
Carbon bisulphide . . . . .	0.260	89	0.0130	0.00339	109
Carbon chloride . . . . .	0.238	98	0.0069	0.00164	225
Steam . . . . .	0.231	100	0.0160	0.00370	100

Temperature limits: for steam, 302° and 110°; for the other vapors, 302° and 68°.

mer therefore varies directly as the cooling water consumption per horse-power, and as the amount of condenser surface necessary. The values of this ratio show no great variation; what difference exists is unfavorable to steam. The following is the comparison for the conditions adopted in Table VII:

TABLE VIII  
CONDENSER SURFACE AND COOLING WATER CONSUMPTION

Vapor.	Area, <i>abcd</i>	Area, <i>cadf</i>	<i>cadf</i> $\div$ <i>abcd</i>	Relative Condenser Surface and Cooling Water Consumption per Horse-power
Alcohol . . . . .	130.0	362.98	2.80	87
Chloroform . . . . .	37.1	112.21	3.02	93
Acetone . . . . .	77.0	235.35	3.06	95
Carbon bisulphide . . . . .	45.9	130.29	2.84	88
Carbon chloride . . . . .	28.3	90.65	3.20	99
Steam . . . . .	255.0	846.66	3.31	102

These commercial factors are represented graphically in Fig. 4.

## X

### The Rankine Cycle

THIS is shown in Fig. 5. Expansion terminates before the pressure has been reduced to that of the exhaust, and the pressure falls at constant volume (line *rs* in both diagrams) at the outer end of the stroke. The heat converted into work is *abc<sub>r</sub>s*: the gross amount of heat expended is, as in the Clausius cycle, *eabcf* =  $h_b - h_a + L_c$ . The efficiency of the former cycle is obviously less than that of the latter.

The area of this cycle may be regarded as the algebraic sum of the quantities of external work done along the three paths *bc*, *cr* and *sa*, which quantities may be denoted by the symbol *W* with appropriate subscripts. Now  $W_{bc} = PV_c$ ,  $W_{sa} = pV_s$ ; and by the common formula for adiabatic expansion,

$$W_{cr} = h_b + r_c - h_u - x_r r_i, \quad . . . . (I)$$

in which expressions *V* denotes the vapor volume at the subscript state and *r* the "internal latent heat of vaporization." Then

$$abc_{r}s = PV_c - pV_s + h_b + r_c - h_u - x_r r_i. \quad . . (J)$$

The conditions of the problem give all quantities excepting  $x_r$ ,  $r_i$  and  $V_s$ . If we assume a limiting temperature at *r*, these also may be readily computed, for  $r_i$  is tabular for a given value of  $t_r$ , and

$$\frac{x_r l_i}{t_r} = x_r n_{ut} = n_b - n_u + n_{bc},$$

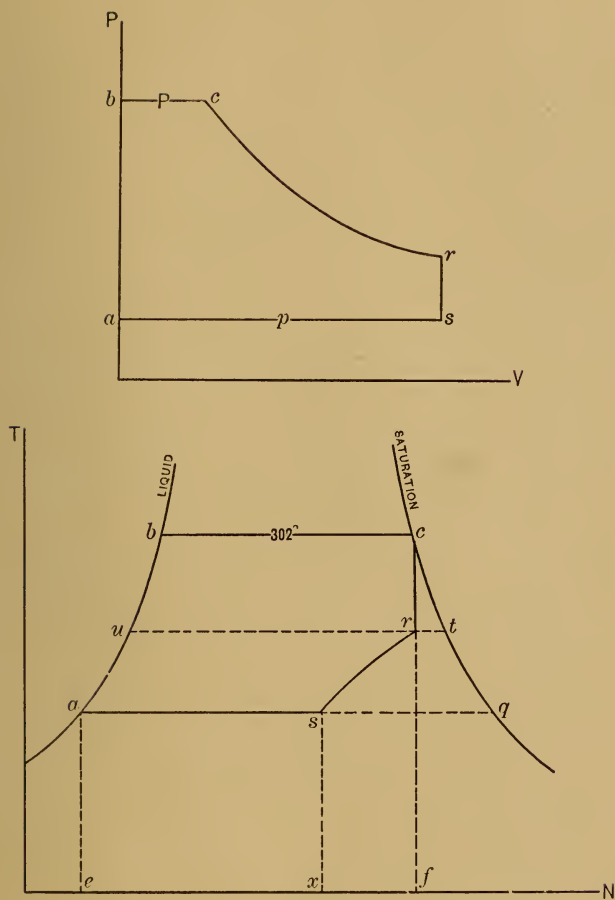


FIG. 5.—Rankine Cycle for Dry Vapor

from which  $x_r$  may be obtained when  $l_t$  is tabular, and

$$V_s = V_r = x_r V_t,$$

very nearly,  $V_t$  being also tabular.

Such a comparison would be of little value. Expansion is in practice limited, not by an assigned temperature  $t_r$ , but by a "ratio of expansion,"  $V_r \div V_c$ , which in simple engines has been established at about the value 4:1, as a compromise between technical cyclic efficiency and the detrimental effect of extreme cylinder condensation at more complete expansions.

This makes the problem more difficult of direct analytic solution, in the absence of knowledge of properties other than the volume and entropy of the wet vapor at the state  $r$ . Such a formula between temperature and volume as is given in Appendix II does not aid us, because it is applicable only to *dry steam*. Both dryness and temperature are unknown at  $r$ , if the ratio of expansion  $V_r \div V_c$  be alone assigned. A carefully plotted entropy diagram, on which the lines of constant volume were drawn at close intervals,<sup>1</sup> would permit of an easy solution; or we might employ simultaneous equations in the forms

$$c \log_e \frac{t_r}{t_a} + \frac{x_r l_t}{t_r} = c \log_e \frac{t_b}{t_a} + \frac{l_c}{t_b},$$

$$V_r = x_r V_t = x_r(f)t_r,$$

latent heats of vaporization and volumes being expressed as functions of temperatures and  $c$  (the specific heat of the liquid) being not too rapidly variable.

The method to be adopted is that suggested by the entropy chart, on which lines of constant volume and constant dry-

<sup>1</sup> See the author's *Applied Thermodynamics* (D. Van Nostrand Co.), 1910, pp. 212, 223.

ness may be drawn. There is no lack of such charts for steam, and it is needless to reproduce one here. Those for other vapors considered have been plotted (Figs. 10 to 13) for this work. The one for ether (Fig. 15) is reproduced, as

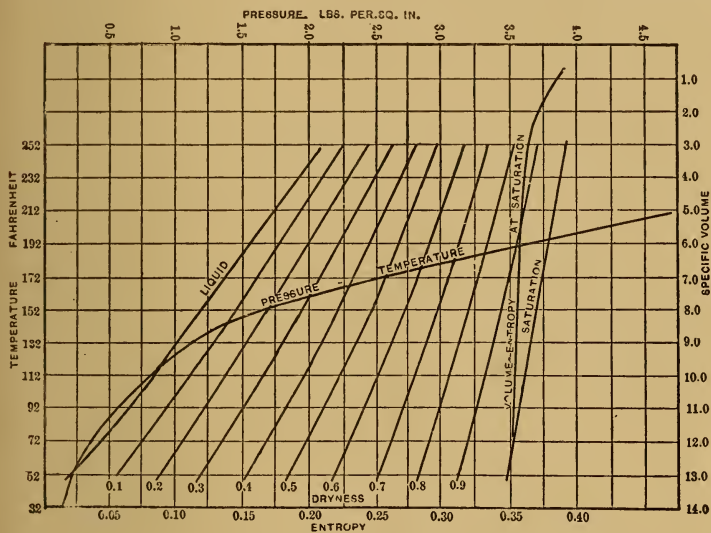


FIG. 15.—Temperature-entropy Diagram for Ether

showing the peculiar behavior of that vapor—evaporation during adiabatic expansion from any initial condition, an evaporation which merges into superheating if expansion from a fairly dry initial condition be sufficiently long continued. Most common vapors condense with adiabatic expansion from an initially dry condition. Carbon chloride (Fig. 13) appears to remain practically in the “just dry” state for even an extreme range of expansion.

## XI

### Efficiencies in the Rankine Cycle; Economical Condenser Temperature

THE cycles compared will be those with initially dry vapor and a ratio of expansion of about 4 to 1. The temperature limits should be, as in the Clausius cycles considered,  $302^\circ$  and  $68^\circ$  for the special vapors and  $302^\circ$  and  $110^\circ$  for steam.

For steam, then,  $V_c=6.28$  and  $V_r$  should be  $6.28 \times 4 = 25.12$ . The entropy diagram gives for  $n_c=1.6319$  and  $V_r=25.12$ ,  $t_r=210^\circ$ . At this temperature,  $n_u=0.3087$ ,  $n_{ul}=1.4510$ . Since  $n_b=0.4398$  and  $n_{bc}=1.1921$ ,

$$0.4398 - 0.3087 + 1.1921 = x_r \times 1.4510,$$

and

$$x_r = 0.912.$$

The steam table gives  $V_t=27.80$ , so that the actual value of  $V_r$  is close to  $0.912 \times 27.80 = 25.4$ , the departure from the assumed value being due to inaccuracy in plotting and reading the entropy chart. We will use the value  $V_r=25.4$ , so that the ratio of expansion will in this case be  $25.4 \div 6.28 = 4.04$ , instead of 4.0, as assumed. Taking the necessary tabular values for substitution in Eq. (J), we have

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (69.03 \times 6.28) - (1.271 \times 25.4) \} \\ &\quad + 271.6 + 828.1 - 177.99 - (0.912 \times 899.0) \\ &= \frac{144}{778} (434 - 32.2) + 271.6 + 828.1 - 177.99 - 820 \\ &= 74.5 + 1099.7 - 997.99 = 176.21 \text{ B.T.U.} \end{aligned}$$



The heat expended is, as in the Clausius cycle, 1101.66 B.T.U., so that the efficiency is  $176.21 \div 1101.66 = 0.16$ .

If we should proceed in this way with the other vapors we should find a total lack of correspondence in the order of efficiencies for the Rankine cycles with that of efficiencies for the Clausius cycles. A rather curious fact, which we are now to consider, will suggest a fairer comparison than that proposed.

Suppose we take the case of steam, working between  $302^\circ$  and  $68^\circ$  F. Values of  $V_r$  and  $x_r$  will be as for the cycle already considered. Then

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (69.03 \times 6.28) - (0.3386 \times 25.4) \} \\ &\quad + 271.6 + 828.1 - 177.99 - (0.912 \times 899) \\ &= \frac{144}{778} (434 - 8.6) + 101.71 = 78.9 + 101.71 = 180.61 \text{ B.T.U.}, \end{aligned}$$

$eabcf = 1143.53$ , as for the Clausius cycle, and

$$\text{Efficiency} = abcrs \div eabcf = 180.61 \div 1143.53 = 0.158.$$

*An increase in temperature range has thus, contrary to expectation, decreased the efficiency of the cycle.* No such result would be possible with the complete-expansion Clausius cycle. Too good a vacuum, with limited expansion, appears to be undesirable.

Fig. 6 suggests an explanation. Let  $abcde$  represent the steam cycle between  $302^\circ$  and  $68^\circ$  F.,  $gbcdf$  that between  $302^\circ$  and  $110^\circ$  F. The additional work,  $agfe$ , of the former cycle, is gained at an expenditure for heat of  $mnga$ . Now,  $mnga = c(t_g - t_a)$ , or, very nearly, 42 B.T.U.: while  $agfe = (P_g - P_a)V_e = \frac{144}{778} \times 25.4 (1.271 - 0.3386) = 4.4$  B.T.U. The

ratio of additional work obtained to additional heat consumed, when the low temperature limit is changed from  $110^\circ$  to  $68^\circ$ , is  $4.4 \div 42 = 0.105$ ; which is less than the efficiency of the  $110^\circ$  cycle, so that the change must necessarily be unprofitable.

Analytically, if  $t_g - t_a$  be small, so that the temperature along the path  $ag$  may be represented by the single symbol

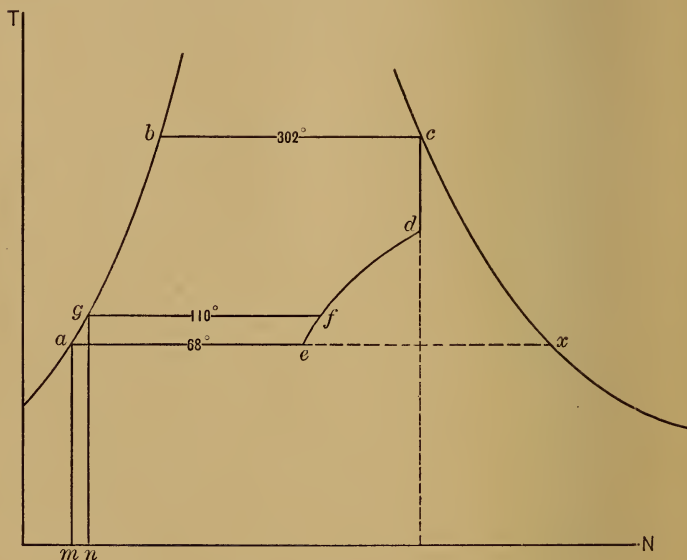


FIG. 6.—Effect of Change in Back Pressure

$t$  (absolute temperature), and  $l$  be the corresponding value of the latent heat of vaporization, the area  $agfe$  is

$$\frac{x_e l}{t} (t_g - t_a).$$

When this is small in relation to the area  $mnga = c(t_g - t_a)$ , or when the quotient

$$\frac{x_e l}{tc}$$

has a lower value than the efficiency of the cycle under consideration; then we may expect to find a lowering of the condenser temperature undesirable, and vice versa. The value of  $x_e$  is, of course, very nearly

$$\frac{4V_c}{V_x}$$

## XII

### Rankine Cycle of Maximum Efficiency

A NEW problem is thus suggested: given the upper temperature,  $t_c$ , and the ratio of expansion  $V_d \div V_c$ , at what lower temperature should the vapor be discharged in order that the efficiency may be a maximum?

Let us take the value  $\frac{x_e l}{t_c}$  as a criterion of the desirable discharge temperature. For steam, with  $4V_c = 25.4$ ,  $c = 1.0$ .

TABLE IX

DESIRABLE CONDENSER TEMPERATURE WITH STEAM  
FOUR EXPANSIONS, FROM 302° F.

Assumed Lower Temperature	$l$	$V_x$	$x_e$	$\frac{x_e l}{t_c}$
68	1053.4	928	0.0274	0.0548
86	1043.4	529.5	0.0480	0.0919
104	1033.4	313.3	0.0811	0.1480
107	1031.7	288.3	0.0879	0.1600

We may infer, therefore, that as the discharge temperature is reduced from 110° to 107°, the efficiency first increases and afterward decreases, passing a maximum at some temperature between these two, and being 0.16 at the two temperatures stated, or practically that at its maximum.

**Alcohol.** This vapor gives  $V_c$ , Fig. 6, as 1.139, so that  $V_d = V_f = V_e = 4 \times 1.139 = 4.556$ . Fig. 10 gives  $t_d = 210^\circ$ ; at which, by interpolation,  $V = 4.73$ . This is the  $V_t$  of Fig. 5, in which, also by interpolation,  $n_u = 0.2073$ ,  $n_{ut} = 0.537$ . Since  $n_b = 0.325$ ,  $n_{bc} = 0.403$ , we have

$$0.325 - 0.2073 + 0.403 = x_r \times 0.537,$$

$$x_r = 0.971,$$

and the check value of  $V_r$  is  $0.971 \times 4.73 = 4.60$ , as against 4.556 intended. Using this in Eq. (J), with  $68^\circ$  as the discharge temperature,

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (142.0 \times 1.139) - (0.86 \times 4.60) \} \\ &\quad + 206.68 + 277.36 - 121.08 - (0.971 \times 331.77) \\ &= \frac{144}{778} (160.4 - 3.90) + 484.04 - 443.08 = 69.98. \end{aligned}$$

The heat expended being 492.98, the efficiency is **0.142**.

The following approximation is now necessary, as in Table IX.

TABLE X  
DESIRABLE DISCHARGE TEMPERATURE WITH ALCOHOL  
FOUR EXPANSIONS, FROM  $302^\circ$  F.

Assumed Lower Temperature	$l$	$V_x$	$x_e$	$\frac{x_e l}{lc}$
86	432.92	91.82	0.0501	0.065
122	420.82	34.20	0.1342	0.135
140	409.73	21.69	0.2120	0.201

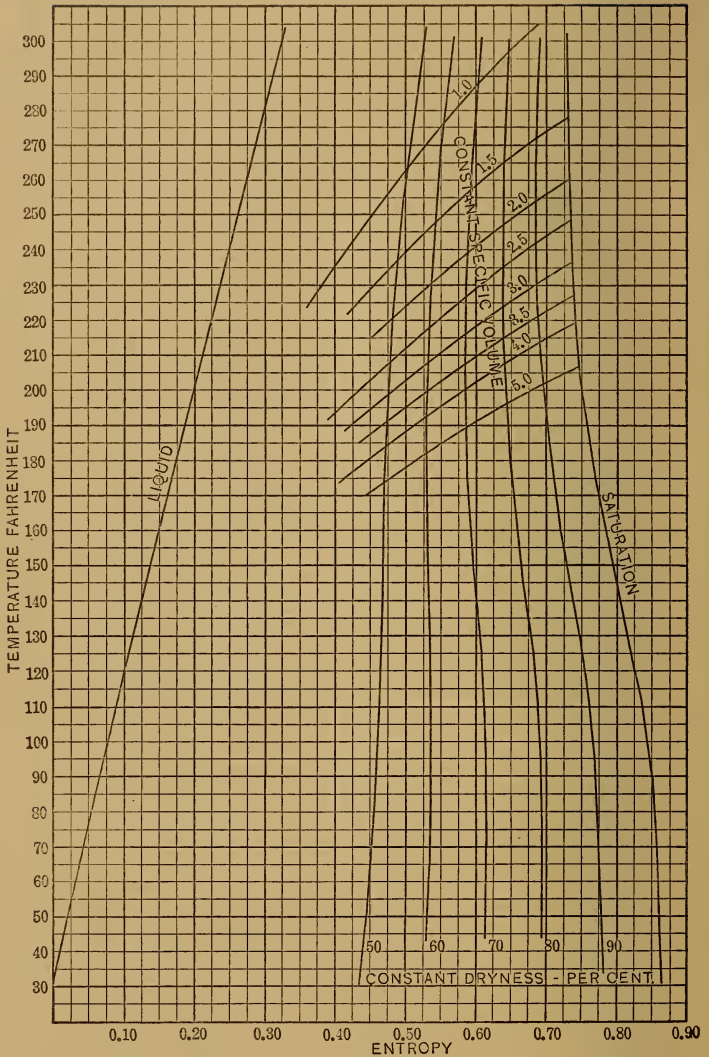


FIG. 10.—Temperature-entropy Chart for Alcohol

For this last lower temperature of 140°, we have

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (142.0 \times 1.139) - (6.78 \times 4.60) \} + 484.04 - 443.08 \\ &= \frac{144}{778} (160.4 - 30.8) + 40.96 = 64.91; \end{aligned}$$

while the difference between the heats of the liquid at 140° and 68° being  $67.27 - 20.56 = 46.71$ , the heat expended is  $492.98 - 46.71 = 446.27$ , and the efficiency is  $64.91 \div 446.27 = 0.145$ . Again, the presumption is that maximum efficiency occurs at some discharge temperature between the two considered, viz., 140° and 68°. To shorten the matter, let us note that

$$abcrs = \frac{144}{778} (160.4 - 4.60P_a) + 40.96.$$

For  $t_a = 122^\circ, 104^\circ, 86^\circ$ , respectively,  $P_a = 4.25, 2.59, 1.52$ , and  $abcrs = 67.06, 68.56, 69.36$ . Also for  $h_a = 54.38, 42.68, 31.48$ , as compared with 20.56 for a 68° discharge temperature, the reductions in heat expenditure are 33.82, 22.12, 10.92, and the respective heat expenditures are 459.16, 470.86, 482.06, giving efficiencies of 0.146, 0.145, 0.144. The discharge temperature had better be 122° than 140°; a lower temperature than 122° is undesirable. Maximum efficiency will be attained when it is between 122° and 140°, probably nearer the former than the latter, and this maximum efficiency will be not far from **0.146**.

**Chloroform.** We have, in Fig. 5,  $V_c = 0.457, V_r = 4 \times 0.457 = 1.828$  (desired value). From the chart, Fig. 11,  $t_r = 181^\circ \text{F}$ . Applying the principle

$$\begin{aligned} n_b - n_u + n_{bc} &= x_r n_{ut}, \\ 0.1077 - 0.0647 + 0.1219 &= 0.1651 x_r, \\ x_r &= 0.998. \end{aligned}$$

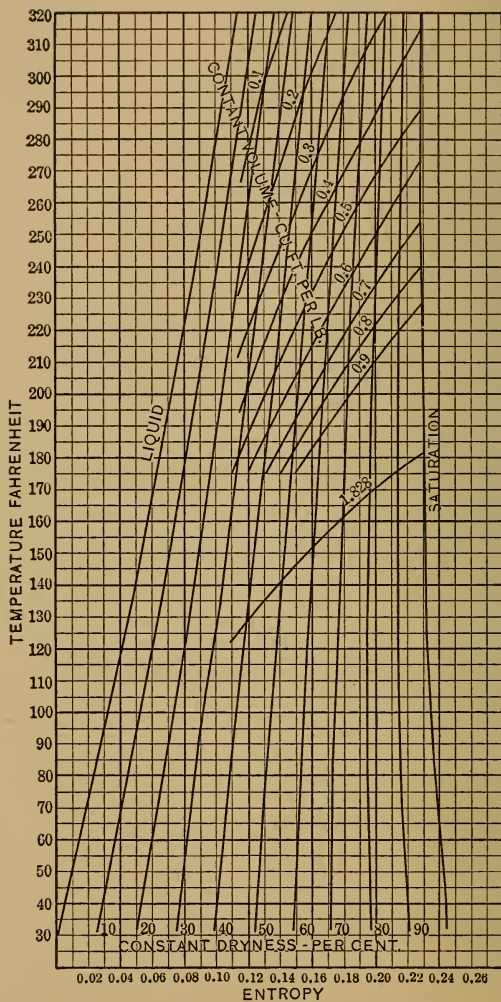


FIG. 11.—Temperature-entropy Chart for Chloroform



From the table,  $V_t=1.85$ , whence  $V_r$  (value employed) checks as  $0.998 \times 1.85 = 1.85$ . By Eq. ( $J$ ), with a discharge temperature of  $104^\circ$ ,

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (141.4 \times 0.457) - (7.14 \times 1.85) \} \\ &\quad + 64.78 + 81.23 - 35.24 - (0.998 \times 95.42) \\ &= \frac{144}{778} (64.4 - 13.2) + 15.67 = 25.17. \end{aligned}$$

The heat expended is 140.85, and the efficiency is  $25.17 \div 140.85 = 0.179$ .

TABLE XI

DESIRABLE DISCHARGE TEMPERATURE WITH  
CHLOROFORMFOUR EXPANSIONS, FROM  $302^\circ \text{F}$ 

Assumed Lower Temperature.	$l$	$V_x$	$x_e$	$\frac{x_e l}{tc}$
86	115.38	10.275	0.1798	0.158
104	113.63	7.1	0.261	0.218

The best condenser temperature is between  $86^\circ$  and  $104^\circ$  and somewhat exceeds **0.179**.

For **acetone**, tabular values above  $284^\circ \text{F}$ ., are extrapolated merely, and the results to be obtained must be regarded with some reserve. The initial volume is about 0.706; that at the end of expansion must then be approximately  $0.706 \times 4 = 2.824$ ; at which, from the chart (Fig. 12), the

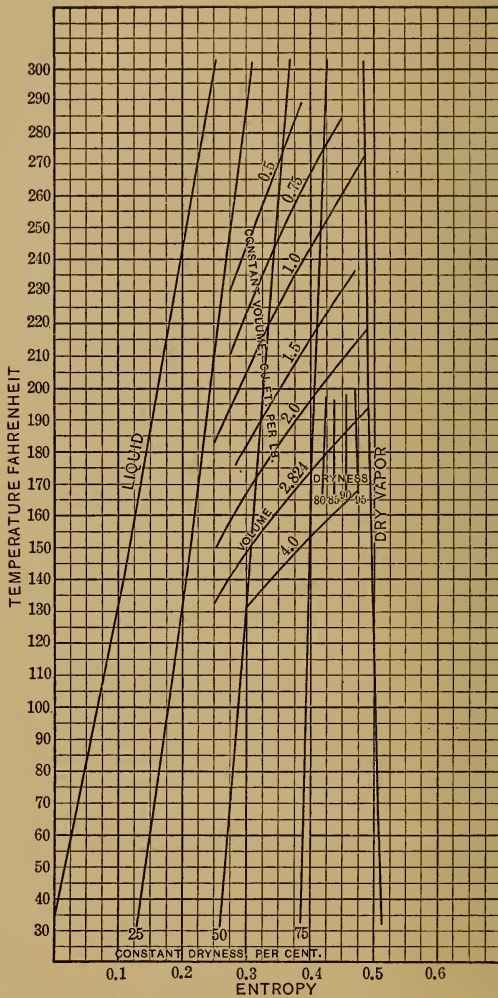


FIG. 12.—Temperature-entropy Chart for Acetone

temperature is  $191^\circ$  and the dryness, 0.975. Then, referring to Fig. 5,

$$n_b - n_u + n_{bc} = n_{ur} = x_r n_{ut},$$

$$0.2465 - 0.1537 + 0.236 = 0.334x_r$$

$$x_r = 0.985 \text{ (accepted value),}$$

and since  $V_t$  is 2.99,  $V_r = 0.985 \times 2.99 = 2.94$ , which value will be employed. For  $t_a = 104^\circ$ , Fig. 5, Eq. (J) gives

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (164 \times 0.706) - (8.12 \times 2.94) \} \\ &\quad + 152.78 + 156.02 - 86.09 - (0.985 \times 196.14) \\ &= \frac{144}{778} (115.7 - 23.7) + 29.21 = 17 + 29.21 = 46.21. \end{aligned}$$

The heat expended is  $h_b + l_{bc} - h_a = 152.78 + 178.13 - 37.60 = 293.31$ , and the approximate efficiency  $46.21 \div 293.31 = 0.158$ .

Now at  $104^\circ$ ,  $l = 240.19$ ,  $c = 0.534$ ,  $V_x$  (Fig. 6) = 13.13,  $x_e = 2.94 \div 13.13 = 0.224$ , and

$$\frac{x_e l}{tc} = \frac{0.224 \times 240.19}{564 \times 0.534} = 0.178,$$

so that a somewhat lower discharge temperature is desirable. If this be  $86^\circ$ ,

$$abcrs = \frac{144}{778} \{ 115.7 - (5.42 \times 2.94) \} + 29.21 = 47.61,$$

the heat expended is  $152.78 + 178.13 - 27.99 = 302.92$ , and the efficiency is in the neighborhood of  $47.61 \div 302.92 = 0.157$ . The inconsistency here is probably due to errors in extrapolated values, so that acetone will be ignored in further comparisons.

**Carbon Chloride.** Here  $V_c=0.510$ ,  $V_r=2.04$  and Fig. 13 gives  $t_r=193^\circ$  with  $x_r$  close to (perhaps slightly above) unity; values which confirm that assumed for  $V_r$ . Eq. ( $J$ ) now gives, if  $t_a=122^\circ$ ,

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (88 \times 0.510) - (6.08 \times 2.04) \} \\ &\quad + 57.11 + 60.78 - 33.18 - 73.40 \\ &= \frac{144}{778} (44.8 - 12.34) + 11.31 = 17.31; \end{aligned}$$

and since  $eabcf = 57.11 + 69.02 - 18.22 = 107.91$ , the efficiency is  $17.31 \div 107.91 = 0.16$ .

At  $t_a=122^\circ$ ,  $l=87.76$ ,  $V=6.554$ ,  $c=0.21$ ,  $x_c$  (Fig. 6)  $= 2.04 \div 6.554 = 0.31$ ; and the efficiency criterion becomes

$$\frac{x_c l}{tc} = \frac{0.31 \times 87.76}{582 \times 0.21} = 0.224.$$

This justifies an investigation for  $t_a=104^\circ$ , at which  $p_a = 4.155$  and

$$abcrs = \frac{144}{778} (44.8 - 8.41) + 11.31 = 18.01,$$

which with  $eabcf = 126.13 - 14.51 = 111.62$  gives an efficiency of  $18.01 \div 111.62 = 0.162$ . Here we have

$$\frac{x_c l}{tc} = \frac{(2.04 \div 9.302) 89.13}{564 \times 0.205} = 0.169;$$

indicating that a still better efficiency will accompany some reduction in discharge temperature. If this be made  $86^\circ$ , however,  $p_a = 2.754$  and

$$abcrs = \frac{144}{778} (44.8 - 5.66) + 11.31 = 18.58;$$

$eabcf = 126.13 - 10.84 = 115.29$ ; and the efficiency is  $18.58 \div 115.29 = 0.161$ .

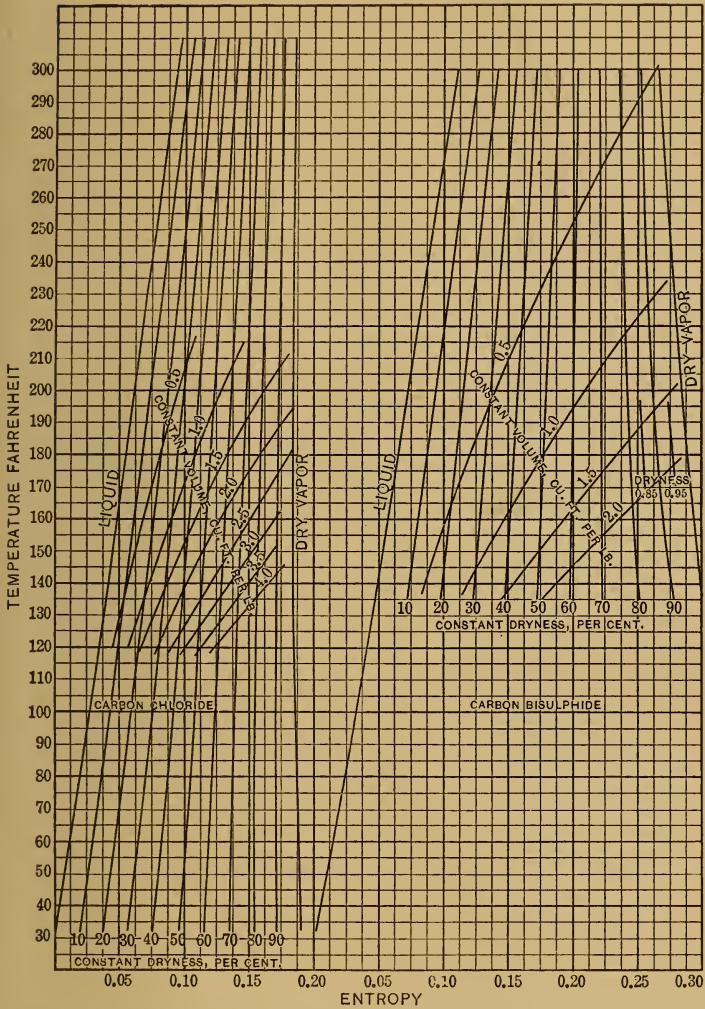


FIG. 13.—Temperature-entropy Charts for Carbon Chloride and Carbon Bisulphide

The best discharge temperature is then between  $86^\circ$  and  $104^\circ$ , and the corresponding efficiency is not far from **0.162**.

Finally, for **carbon bisulphide**,  $V_c$  (Fig. 5) = 0.502,  $V_r = 2.008$ , and Fig. 13 gives  $t_r = 173^\circ$ ,  $x_r = 0.90$ . From the table,  $V_l = 2.226$ , whence  $V_r$  (actual value used) =  $2.226 \times 0.90 = 2.003$ . If we take  $t_a$  at  $68^\circ$ ,

$$\begin{aligned} abcrs &= \frac{144}{778} \{ (176 \times 0.502) - (5.76 \times 2.003) \} \\ &\quad + 66.82 + 101.9 - 34.07 - (0.90 \times 128.44) \\ &= \frac{144}{778} (88.3 - 11.56) + 19.05 = 33.25. \end{aligned}$$

Here  $eabcf = 66.82 + 117.90 - 8.53 = 176.19$ , and the efficiency is  $33.25 \div 176.19 = 0.189$ . Applying the criterion,

$$\frac{x_{cl}}{tc} = \frac{(2.003 \div 12.879) \times 158.44}{528 \times 0.2385} = 0.196,$$

so that in this case maximum efficiency (which will not much exceed **0.189**) will be obtained at a discharge temperature possibly a little below  $68^\circ$  F.

We now tabulate these results:

TABLE XII  
MAXIMUM EFFICIENCIES WITH FOUR EXPANSIONS  
INITIALLY DRY VAPORS, FROM  $302^\circ$  F.

Vapor	Discharge Temperature	Efficiency	Vacuum (Inches of Mercury)	Order of Efficiencies
Alcohol . . . . .	$122^\circ$ - $140^\circ$	0.146	16.13 to 21.26	5
Chloroform . . . . .	$86^\circ$ - $104^\circ$	0.179+	15.38 to 20.17	2
Carbon bisulphide	below $68^\circ$	0.189+	at $68^\circ$ , 18.19	1
Carbon chloride . .	$86^\circ$ - $104^\circ$	0.162	21.46 to 24.32	3
Steam . . . . .	$107^\circ$ - $110^\circ$	0.16	27.33 to 27.55	4

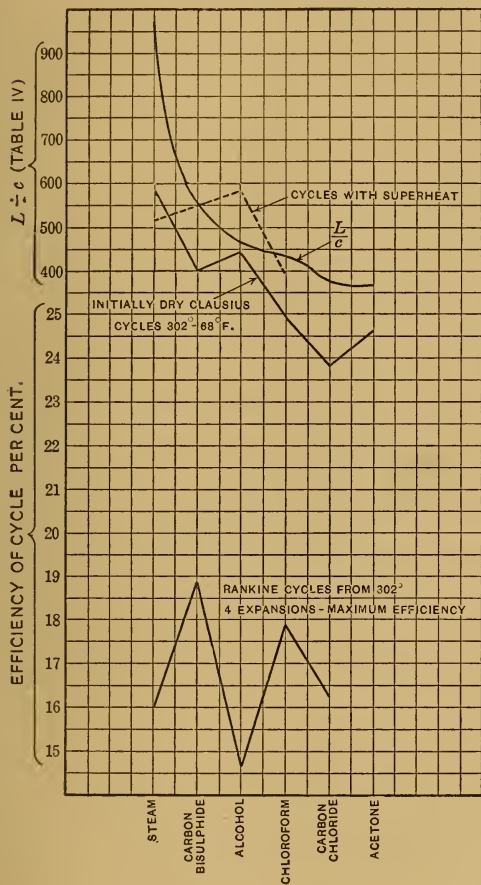


FIG. 14.—Cyclic Efficiencies and Criterion

The order of efficiencies is strikingly different from that for those cycles in which expansion is complete. It is noteworthy also that with some of the vapors the best efficiency is attained with only a moderate degree of vacuum. Alcohol is the only vapor showing a lower efficiency than steam; those with the other vapors are such as to justify the expectation of saving from 1 to 18 per cent of the fuel by their substitution for steam.

In compound condensing engines, with ratios of expansion greatly exceeding 4, the most economical discharge temperature would probably be the lowest attainable, and the efficiencies of the various vapors would rank more nearly in the order found for the Clausius cycle.

The use of the  $\frac{T-t}{T}$  criterion furnished by the Carnot cycle is wholly unreliable; but it is a curious fact that in Table XII the efficiencies rank very nearly in the order of the temperature ranges.

Some graphical expressions for both the Clausius and Rankine cycle results are given in Fig. 14.



## XIII

### Commercial Factors with the Rankine Cycle

IF we apply the principles already enunciated for the cycles of complete expansion, we have, in Fig. 5,

$\frac{abcrs}{eabcf}$  = efficiency, as an inverse measure of the relative boiler capacities necessary;

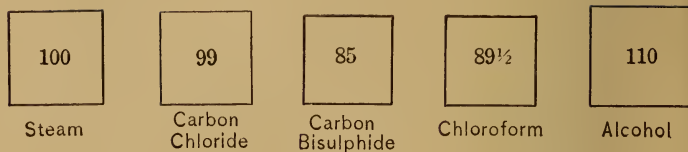
$\frac{abcrs}{eabcf}$   $\times$  *volume of liquid*, as an inverse measure of the relative "times to get up steam";

$\frac{easrf}{abcrs} = \frac{eabcf - abcrc}{abcrs}$ , as a measure of the relative amounts of condenser surface and cooling water necessary;

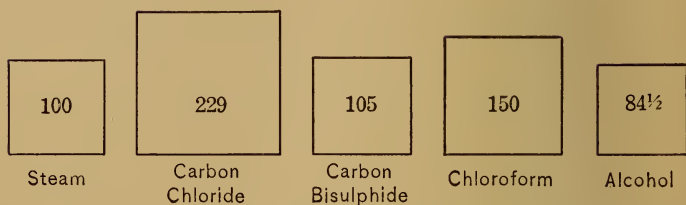
and (a new feature)

$\frac{V_r = V_s}{abcrs}$ , as a measure of the relative *sizes of cylinder necessary for a given output*.

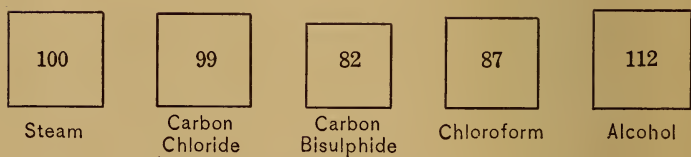
The comparisons are shown graphically in Fig. 7. Alcohol is an unattractive vapor on account of its low efficiency. Carbon bisulphide presents the unusually desirable features



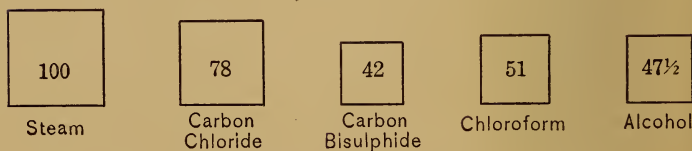
Relative Boiler Capacities Necessary



Relative Times to "Get Up Steam" in the Same Boiler



Relative Amounts of Condenser Surface and Cooling Water



Relative Sizes of Cylinders

FIG. 7.—Comparative Proportions of Power Plants Using Various Fluids in the Rankine Cycle from 302° F., with Four Expansions. All Developing the Same Horse-power

of highest efficiency, maximum cylinder capacity, minimum condenser surface and cooling water consumption, minimum boiler capacity; and a "time to get up" pressure only 5 per cent greater than is necessary with steam.

TABLE XIII

BOILER CAPACITY AND STEAMING RATE

RANKINE CYCLES, FOUR EXPANSIONS, DRY VAPOR FROM 302° F.

Vapor	Efficiency	Relative Boiler Capacity Necessary	Volume of Liquid	Volume × Efficiency	Relative Time to "Get Up Steam"
Alcohol . . . . .	0.146	110	0.0208	0.00304	84½
Chloroform . . . . .	0.179	89½	0.0096	0.00172	150
Carbon bisulphide . . . . .	0.189	85	0.0130	0.00245	105
Carbon chloride . . . . .	0.162	99	0.0069	0.00112	229
Steam . . . . .	0.16	100	0.0160	0.00256	100

TABLE XIV

CONDENSER SURFACE AND COOLING WATER CONSUMPTION

RANKINE CYCLES, FOUR EXPANSIONS, DRY VAPOR FROM 302° F.

Vapor	<i>abcrs</i>	<i>ebcf</i>	<i>earf</i>	<i>earf</i> ÷ <i>abcrs</i>	Relative Condenser Surface and Cooling Water Consumption
Alcohol . . . . .	67.06	459.16	392.10	5.85	112
Chloroform . . . . .	25.17	140.85	115.68	4.58	87
Carbon bisulphide	33.25	176.19	142.94	4.30	82
Carbon chloride . .	18.01	111.62	93.61	5.20	99
Steam . . . . .	176.21	1101.66	925.45	5.25	100

TABLE XV

## SIZE OF CYLINDER FOR A GIVEN OUTPUT

RANKINE CYCLES, FOUR EXPANSIONS, DRY VAPOR FROM 302° F.

Vapor	$abcrs$	$V$	$V_r \div abcrs$	Relative Volume of Cylinder
Alcohol.....	67.06	4.60	0.0685	47½
Chloroform.....	25.17	1.85	0.0735	51
Carbon bisulphide....	33.25	2.003	0.0602	42
Carbon chloride.....	18.01	2.04	0.1125	78
Steam.....	176.21	25.4	0.1440	100

# XIV

## Rankine Cycles with Superheat

NOT enough is known of the properties of these vapors when in the superheated condition to warrant the attempt to solve comparative cycles of this type. Even for steam,

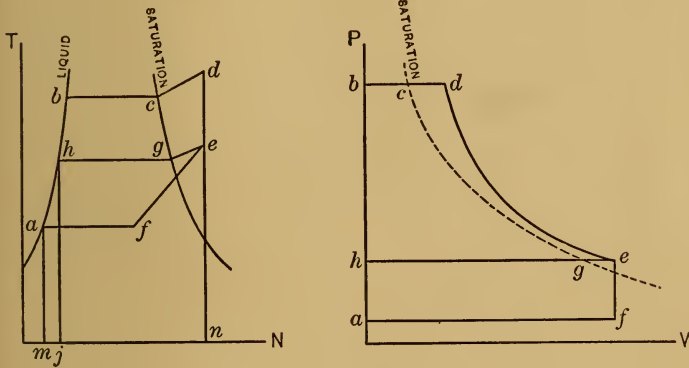


FIG. 8.—The Rankine Cycle with Superheat

the last word has probably not been said on such properties. The following method for computing the efficiency etc., of any Rankine cycle with the vapor initially superheated, is believed to be accurate and perfectly simple: requiring only exact values for the specific heats and entropies calculated therefrom. The need of the method arises from the

inaccuracy in computing the work along an even partially superheated adiabatic, either by the  $\frac{pv-PV}{n-1}$  formula or by an expression for the loss of internal energy. The present method may be considerably shortened by the employment of the Mollier or total-heat entropy diagram.

CASE I Let expansion be wholly in the superheated region, the steam becoming saturated (dry or wet) during the terminal drop - cycle  $abcdef$ , Fig. 8. Then

$$\text{Efficiency} = \frac{\text{Work}}{\text{Heat expended}} = \frac{abcdef}{mabcdn}.$$

Draw the line of constant pressure  $egh$  through  $e$ . Then

$$\begin{aligned} \text{Efficiency} &= \frac{hbcdeg + ahgef}{mabcdn} \\ &= \frac{(jhbcdn - jhgen) + \frac{144}{778} \{ (P_h V_e) - (P_a V_e) \}}{mabcdn} \\ &= \frac{H_a - h_h - H_e + h_h + \frac{144}{778} (P_h - P_a) V_e}{H_a - h_a} \\ &= \frac{H_a - H_e + \frac{144}{778} V_e (P_h - P_a)}{H_a - h_a}, \end{aligned}$$

$H_a$  and  $H_e$  being the total heats above  $32^\circ$  F., at the states denoted by their subscripts,  $h_a$  the heat of the liquid at  $a$ , and  $V$  and  $P$  pressures and volumes. The entropy and volume at  $e$  determine the total heat and pressure at that point.

CASE II. If the vapor remains superheated at the end of the terminal drop (i.e., at the point  $f$ ), the computation is unaltered, and the fact of such superheat might even be unsuspected.

CASE III. If the vapor becomes saturated during expansion, its initial entropy must have been less than that of the dry vapor at the terminal pressure. This is the most probable case, and the condition is sure to be detected when the total heat is ascertained at  $e$ . The saturated steam tables will give  $H_e$ , and the expression for efficiency is not changed.

## XV

### Summary: Conclusions

THE use of a special vapor to replace steam might be justified on one of three grounds:

(a) *A reduced lower temperature limit for the cycle without the necessity for an impracticably high vacuum.* The extreme limit is determined, however, by the cooling water supply, and the gain in this direction appears likely, from a rigid application of the second law of thermodynamics, to be small.

(b) *But the properties of the substitute vapor may be such as to cause a greater gain than is thus indicated.* Examination shows that steam ranks best as to the critical ratio

$$\frac{\text{Latent heat of vaporization}}{\text{Specific heat of liquid}},$$

between certain assumed temperature limits, at which, correspondingly, it gives the highest efficiency. These limits (302° and 68° F.) are impracticable for steam, though practicable with the other vapors. With a more practicable lower limit of 110° F. for steam, it gives with complete expansion an efficiency below that attainable by the other (saturated) vapors. This comparison is of practical importance only with the turbine engine. The velocities attained by complete adiabatic expansion between the assumed



limits are with the substitute vapors in all cases much less than those attained with steam.

In cycles with terminal drop, at such ratios of expansion as are common in simple engines, all of the vapors except alcohol surpass steam in efficiency, but this superiority is not traceable to a reduced lower temperature limit. This limit is sometimes too low for best efficiency in the simple condensing engine. The limit at which the efficiency is a maximum may be approximated from the variation in the determining ratio

$$\frac{x_e l}{tc}$$

Maximum efficiencies for the various saturated vapors in this type of cycle occur at lower temperature limits ranging all the way from 68° to 140° F.

The total unreliability of any surmises based on the Carnot expression,

$$\frac{T-t}{T},$$

is evident. It may be objected that the use of a uniform expansion ratio of 4:1 in all cases is an improper assumption: that in a vapor showing relatively slight—or no—condensation with adiabatic expansion, the influence of cylinder condensation would be so mitigated that the ratio of expansion might be advantageously increased. But cylinder condensation means virtually heat transfer; and this heat transfer would go on just the same as long as the substance remained a wet vapor. Further, the evils of such transfer are largely evidenced in *initial* condensation; that which occurs, not during expansion, but during *admission* of steam to the cylinder. A given loss of heat to the walls actually means a greater loss of dryness in the case of the substitute vapors,

because the heat contents of given weights of such vapors are less than those of the same weight of steam. This may appear an argument in favor of the use of a *lower* ratio of expansion in their case: but, on the other hand, the substitute vapors uniformly contain more heat and give more work, in proportion to the *space* which they occupy.

(c) *The capacity of the apparatus may be affected by the properties of the fluid chosen.* It appears that there are perceptible advantages with some of the substitute vapors in respect to boiler capacity, time of getting into operation, condenser capacity and amount of cooling water necessary, as long as we limit the consideration to the complete expansion type of cycle. With the terminal drop cycle, the vapors maintain their advantage in all respects excepting that of "quick steaming": and they produce from 50 to 75 per cent more power from a cylinder of given size than does steam.

The objections to the use of a substitute fluid include:

(a) *Its cost.* This need not be prohibitive, if the leakage loss is not excessive in proportion to the gain of efficiency.

(b) *Increased maximum pressure.* This is associated with all of the fluids, with the possible exception of carbon chloride at high temperatures. This substance comes nearest to the desired pressure-temperature relation, standing to steam in much the same relation as it does to alcohol at a lower temperature. Its pressure-temperature curve is of abnormally slight slope.

The binary vapor principle permits of a slight gain without excessive maximum pressure; but involves more complication than would the use of a substitute vapor. In practice, pressure conditions influence the cyclic range and superheating may be resorted to in order to increase the range. With superheat, a high specific heat of the superheated vapor and a left-hand location for the pressure-temperature curve (Fig. 2) furnish criteria of desirability.

The disadvantage of an increased maximum pressure may be offset by the reduction in size of cylinder probable with all of the substitute fluids. The only remaining question is, then, whether with such a fluid a sufficient increase in efficiency may be obtained to offset the expense due to leakage. With superheat and complete expansion the answer appears to be in the negative. With only saturated vapor employed, we have found at least one condition at which, with carbon bisulphide, for example, leakage amounting to the percentage

$$\frac{.18}{a},$$

might be tolerated,  $a$  representing the ratio of the cost of carbon bisulphide, pound for pound, to that of coal.

The properties of some of the vapors are not known with great exactness, and the figures presented are in all cases approximate. Investigation of terminal drop cycles with, say, 16 expansions, both saturated and superheated, is warranted; but on the whole it seems safe to say that there is nothing inherently absurd in the proposal to use some vapor other than steam for power production. The substitution appears far more promising than the use of a binary vapor on the steam cylinder exhaust.



## APPENDICES

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### APPENDIX I

#### The Vapors Discussed

THE alcohol referred to ( $C_2H_6O$ ), is the ordinary ethyl alcohol (not wood alcohol); a light colorless, inflammable, rather pleasant-smelling liquid. When free from water its specific gravity is 0.785. It boils at  $172^\circ F.$ , and has been frequently used as a working fluid in heat engines.

**Chloroform** ( $C_2HO_3$ ), known from its use as an anæsthetic, is a heavy clear fluid of powerful odor, specific gravity about 1.48, boiling-point  $140-144^\circ F.$  The commercial product sells for about 25 cents a pound.

**Acetone** ( $C_3H_6O$ ), is a colorless liquid of specific gravity 0.797 and boiling-point  $135^\circ F.$

**Carbon Bisulphide** ( $CS_2$ ), costs (in a somewhat impure state) about 4 cents a pound. It is a poisonous pungent-smelling clear liquid, boiling at  $115^\circ F.$  The specific gravity is 1.27.

**Carbon Chloride** ( $CCl_4$ , the tetrachloride), boils at  $168-171^\circ F.$ , is 1.6 times as heavy as water, and costs about 8 cents a pound. It has recently been employed as a cleansing fluid in place of gasoline. It is claimed that it can be manufactured on a large scale at a cost much below the

present price. The ordinary commercial substance is a transparent fluid, with an odor suggesting garlic. It is slowly hydrolized by water, forming  $\text{CO}_2$  and  $\text{HCl}$ .

These fluids, with ether, gasoline and 90 per cent benzol, are all grease solvents; most of them are inflammable, but in this respect chloroform and carbon chloride are exceptions. All seem to be non-corrosive in their action on iron pipes or plates. Reference should be made to the paper by Booth, "Commercial Extraction of Greases and Oils," *Trans. Am. Inst. Chem. Engrs.*, II, 1909, 248: and to p. 114 of Gill's "Oil Analysis," relating to the action of oils on metals.

## APPENDIX II

### The Volume Temperature Relation of Dry Steam

THE sources from which a relation between the volume and temperature of saturated steam must be found are, essentially, the exponential equations of Rankine and Zeuner for the pressure-volume relation,

$$PV^n = \text{constant},$$

where  $P$  and  $V$  are corresponding specific pressures and volumes and  $n$  is either  $\frac{1}{8}$  (Rankine) or 1.0646 (Zeuner): and the Thiesen formula for pressure-temperature,

$$t \log \frac{P}{14.7} = 5.409(T - 212) - 8.71 \times 10^{-10}[(689 - T)^4 - 477^4],$$

in which  $t$  is the absolute and  $T$  the Fahrenheit temperature, and  $P$  is in pounds per square inch. The pressure-volume formula is an empirical expression intended to describe the results following the application of the well-known Clapeyron differential equation from which specific volumes are usually calculated. The pressure-temperature expression is also empirical, but stands on a somewhat more satisfactory footing, expressing the results of recent experimental work so closely that it has been used in computing the lately published steam tables of Marks and Davis. It is, however, too cumbersome for our purpose, which is that

of deriving a fairly accurate and quickly available expression for the relation between volume and temperature.

We will adopt the Marks and Davis tables (Longmans, Green & Co., 1909) for reference. A recent magazine article (*Power*, March 8, 1910), gives the surprisingly accurate expression,

$$t = 200p^{\frac{1}{6}} - 101,$$

for pressure (pounds per square inch) and temperature Fahrenheit. This gives confirmation of tabular values with an error not exceeding that involved in computation with a 10-inch slide rule. If we combine this equation with that between pressure and volume, we find

$$p^{\frac{1}{6}} = \frac{t+101}{200} = 0.005t + 0.505,$$

$$pv^{\frac{17}{6}} = (0.005t + 0.505)^6 v^{\frac{17}{6}} = \text{a constant, } 477,$$

the approximate evaluation of which is as follows:

$t$	$0.005t + 0.505$	$\log(I)$	$6\log(I)$	$I^6$	$v^{\frac{17}{6}}$	$\log v^{\frac{17}{6}}$	$\frac{1}{17} \log v^{\frac{17}{6}} = \log v$	$v$
101.83	1.01415	0.0061	0.0366	1.089	437.5	2.6418	2.49	309
153.01	1.270	0.104	0.624	4.2	113.5	2.056	1.934	85.9
182.86	1.419	0.152	0.912	8.16	58.4	1.766	1.661	45.8
200	1.505	0.178	1.068	11.69	40.9	1.612	1.516	32.8
220	1.605	0.206	1.236	17.2	27.9	1.446	1.359	22.82
240	1.705	0.232	1.392	24.62	19.36	1.2877	1.211	16.23
260	1.805	0.2565	1.5390	34.58	13.8	1.1402	1.072	11.8
280	1.905	0.28	1.68	47.9	9.97	0.998	0.938	8.66
300	2.005	0.3023	1.8138	65.04	7.315	0.864	0.812	6.49
320	2.105	0.324	1.944	87.9	5.43	0.735	0.691	4.91
340	2.205	0.344	2.064	115.8	4.125	0.616	0.58	3.8
401.1	2.5105	0.4	2.4	251	1.9	0.28	0.264	1.835



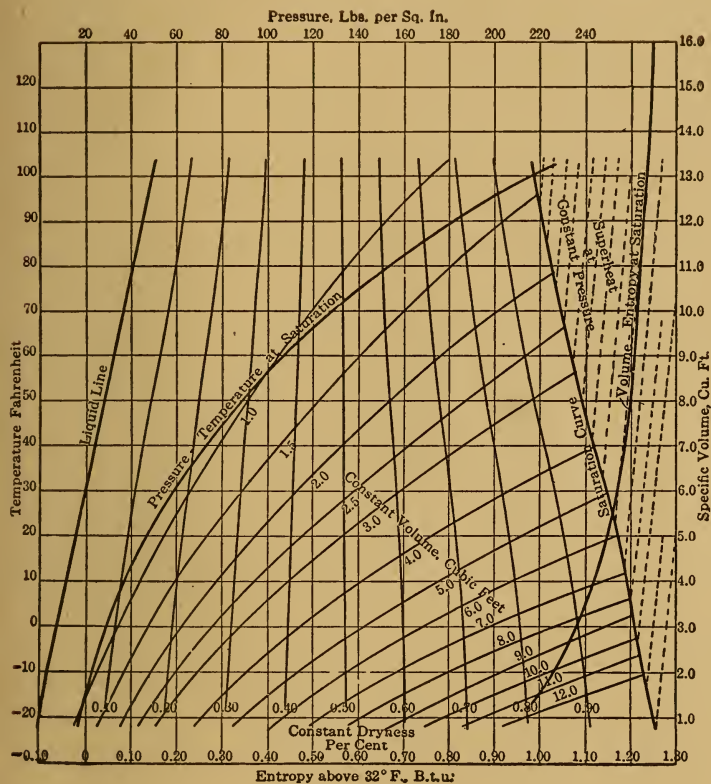


FIG. 16.—Temperature-entropy Diagram for Ammonia

The values of  $v$  closely correspond with those in the steam table adopted, excepting in the case of the lower temperatures—below  $200^\circ$ . A simpler formula is, however, desirable.

If we assume the possibility of an expression in the form  $tv^n = \text{constant}$ , then by successive trials we may find the most plausible value of  $n$  to be 0.248, and

$$tv^{0.248} = 477\frac{1}{2}.$$

This gives the following approximate results.

$v$	$\log v$	$0.248 \log v$	$v^{0.248}$	$t$
30	1.4776	0.3663	2.323	206
25	1.398	0.3465	2.22	216
20	1.3016	0.3233	2.102	227
15	1.176	0.2919	1.958	244
10	1.0	0.248	1.77	270
8	0.936	0.2315	1.701	281
5	0.6985	0.1725	1.488	321

Between the tabulated limits, this has an accuracy within one or two degrees; but it is wholly unreliable for either very low or very high temperatures. Fortunately, it is the *medium* temperatures (between  $200^\circ$  and  $260^\circ$ ) that we are principally concerned with; and within this range the temperature varies very nearly *inversely as the fourth root of the specific volume*—a convenient statement in the absence of logarithms. The fact that both expressions given have about the same constant term is a curious coincidence.

In Fig. 9, the tabular volume-temperature curve has been plotted along with that given by the latter of the two equa-

tions. There would be no purpose served by plotting the more accurate values from the former equation, which within the charted limits, and to the scale adopted, actually coincide

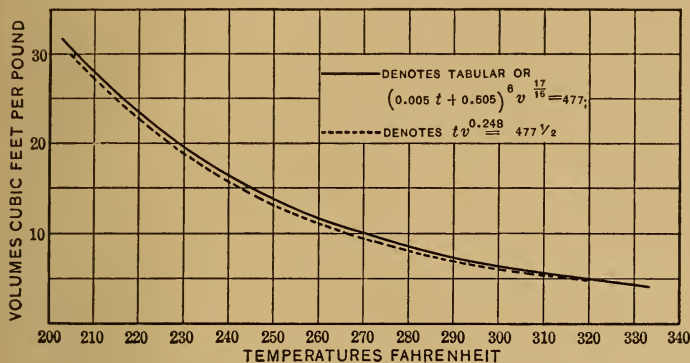


FIG. 9.—Temperature-volume Curves of Dry Steam

with those of the tabular curve, the departure from coincidence being less than 1 per cent even at a temperature above  $400^{\circ}$ . The error rapidly increases, however, as the temperature is lowered below  $200^{\circ}$ , being about 3 per cent at  $183^{\circ}$ , 5 per cent at  $153^{\circ}$  and 7 per cent at  $102^{\circ}$ .

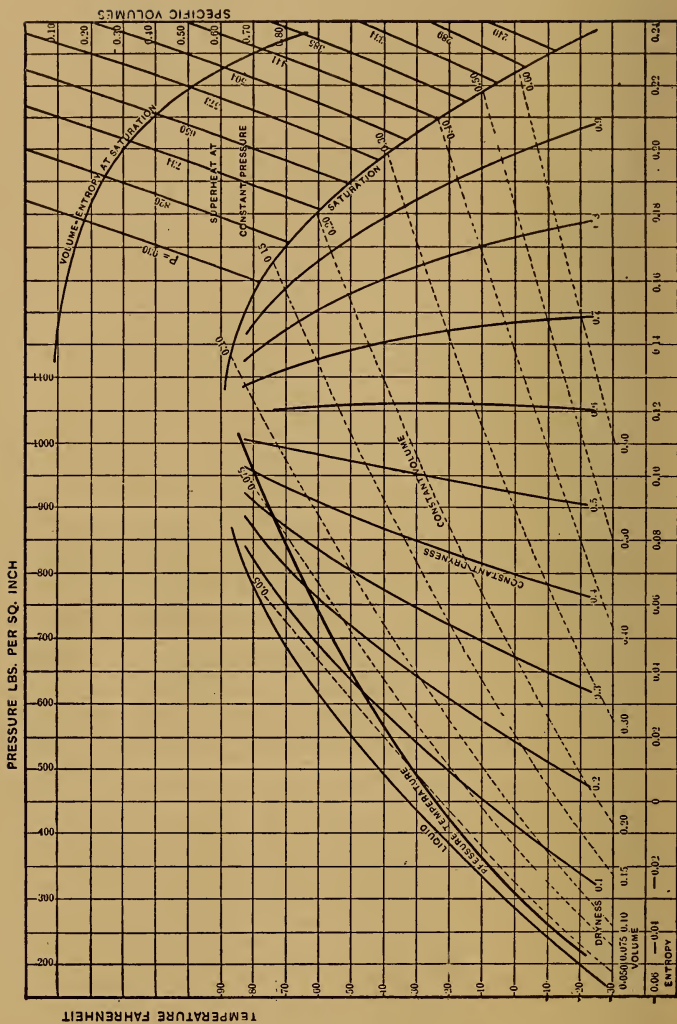


Fig. 17.—Temperature-entropy Diagram for Carbon Dioxide

TABLE XVI.—PROPERTIES OF THE DRY SATURATED VAPOR OF ALCOHOL

NOTE. Tables XVI to XX are abstracted by permission from Klein's Translation of Zeuner's *Technical Thermodynamics*. (D. Van Nostrand Co.)

Temperature, Fahrenheit	Heat of the Liquid above 32°, B.T.U.	Latent Heat of Vaporization, B.T.U.	Internal Latent Heat of Vaporization, B.T.U.	Specific Volume of Dry Vapor— Specific Volume of Liquid, Cu.Ft.
32	0.00	425.70	402.18	513.989
50	10.06	429.86	405.62	277.595
68	20.56	433.04	407.90	156.956
86	31.48	432.92	406.95	91.800
104	42.68	428.92	402.29	55.289
122	54.38	420.82	393.74	34.174
140	67.27	409.73	382.38	21.671
158	80.24	397.12	369.60	14.112
176	93.80	383.56	355.94	9.430
194	107.95	370.85	343.08	6.479
212	122.72	358.42	330.49	4.564
230	138.13	347.15	318.97	3.305
248	154.21	336.29	307.85	2.443
266	170.96	325.84	297.10	1.845
284	188.46	316.44	287.31	1.424
302	206.68	306.86	277.36	1.118

TABLE XVII.—SATURATED VAPOR OF CHLOROFORM

32	0.00	120.60	112.45	37.899
50	4.19	118.88	110.36	23.536
68	8.41	117.14	108.29	15.314
86	12.64	115.38	106.23	10.265
104	16.87	113.63	104.20	7.090
122	21.13	111.84	102.15	5.025
140	25.42	110.03	100.11	3.646
158	29.72	108.20	98.05	2.702
176	34.04	106.36	96.00	2.042
194	38.38	104.49	93.92	1.571
212	42.73	102.62	91.85	1.230
230	47.11	100.71	89.75	0.977
248	51.50	98.80	87.65	0.788
266	55.91	96.86	85.52	0.644
284	60.34	94.91	83.38	0.533
302	64.78	92.94	81.23	0.447
320	69.25	90.95	79.05	0.378

TABLE XVIII.—SATURATED VAPOR OF ACETONE

Temperature, Fahrenheit	Heat of the Liquid above 32°, B.T.U.	Latent Heat of Vaporization, B.T.U.	Internal Latent Heat of Vaporization, B.T.U.	Specific Volume of Dry Vapor— Specific Volume of Liquid, Cu.Ft.
32	0.00	252.90	237.34	68.206
50	9.18	250.22	233.19	42.851
68	18.52	247.20	228.96	28.043
86	27.99	243.86	224.76	18.932
104	37.60	240.19	220.38	13.111
122	47.36	236.19	215.82	9.286
140	57.26	231.87	211.06	6.707
158	67.30	227.22	206.05	4.937
176	77.49	222.23	200.77	3.696
194	87.82	216.92	195.23	2.811
212	98.30	211.26	189.38	2.171
230	108.90	205.31	183.29	1.700
248	119.66	199.01	176.90	1.347
266	130.57	192.39	170.25	1.081
284	141.61	185.43	163.29	0.876

TABLE XIX.—SATURATED VAPOR OF CHLORIDE OF  
CARBON

32	0.00	93.60	87.40	52.196
50	3.58	92.61	86.16	31.990
68	7.18	91.57	84.86	20.465
86	10.84	90.37	83.42	13.569
104	14.51	89.13	81.94	9.295
122	18.22	87.76	80.34	6.547
140	21.96	86.33	78.70	4.729
158	25.74	84.78	76.97	3.489
176	29.56	83.12	75.15	2.624
194	33.39	81.40	73.29	2.006
212	37.26	79.56	71.35	1.554
230	41.17	77.65	69.36	1.219
248	45.11	75.62	67.29	0.966
266	49.09	73.49	65.15	0.772
284	53.08	71.30	63.00	0.622
302	57.11	69.02	60.78	0.503
320	61.20	66.60	58.47	0.408

TABLE XX.—SATURATED VAPOR OF BISULPHIDE OF CARBON

Temperature, Fahrenheit	Heat of the Liquid above 32°, B.T.U.	Latent Heat of Vaporization, B.T.U.	Internal Latent Heat of Vaporization, B.T.U.	Specific Volume of Dry Vapor— Specific Volume of Liquid, Cu.Ft.
32	0.00	162.00	149.02	28.172
50	4.25	160.31	146.90	18.762
68	8.53	158.44	144.62	12.866
86	12.83	156.39	142.20	9.058
104	17.17	154.15	139.63	6.526
122	21.53	151.76	136.93	4.801
140	25.94	149.16	134.06	3.599
158	30.35	146.41	131.07	2.742
176	34.81	143.46	127.91	2.123
194	39.29	140.35	124.63	1.666
212	43.81	137.05	121.19	1.323
230	48.35	133.58	117.62	1.064
248	52.92	129.92	113.89	0.863
266	57.53	126.09	110.03	0.708
284	62.15	122.10	106.05	0.586
302	66.82	117.90	101.90	0.489



TABLE XXI.—ENTROPY TABLES

Temperature Fahrenheit	Alcohol			Chloroform			Acetone			Carbon Bisulphide			Carbon Chloride		
	$n_w$	$n_e$	$n_s$	$n_w$	$n_e$	$n_s$	$n_w$	$n_e$	$n_s$	$n_w$	$n_e$	$n_s$	$n_w$	$n_e$	$n_s$
32	—	0.865	0.865	—	0.2448	0.2448	—	0.514	0.514	0.514	0.329	0.329	—	0.19	0.19
50	0.0198	0.842	0.862	0.0087	0.2350	0.2437	0.0185	0.491	0.510	0.510	0.314	0.323	0.0073	0.1816	0.1889
68	0.0402	0.820	0.860	0.0172	0.2220	0.2392	0.0368	0.469	0.506	0.506	0.3	0.317	0.0145	0.1733	0.1878
86	0.0600	0.793	0.853	0.0257	0.2108	0.2365	0.0549	0.446	0.501	0.501	0.286	0.312	0.0215	0.1652	0.1867
104	0.0800	0.760	0.840	0.0335	0.2017	0.2352	0.0730	0.426	0.499	0.499	0.274	0.307	0.0284	0.1581	0.1865
122	0.1006	0.724	0.825	0.0412	0.1920	0.2332	0.0910	0.406	0.497	0.497	0.260	0.301	0.0350	0.1509	0.1860
140	0.1233	0.683	0.806	0.0486	0.1837	0.2323	0.1080	0.386	0.494	0.494	0.249	0.297	0.0416	0.1436	0.1852
158	0.1447	0.642	0.787	0.0558	0.1752	0.2310	0.1244	0.369	0.493	0.493	0.237	0.293	0.0479	0.1371	0.1850
176	0.1658	0.603	0.769	0.0628	0.1671	0.2299	0.1404	0.350	0.490	0.490	0.226	0.289	0.0541	0.1308	0.1849
194	0.1875	0.566	0.754	0.0697	0.1600	0.2297	0.1565	0.331	0.488	0.488	0.215	0.285	0.0602	0.1244	0.1846
212	0.2098	0.533	0.743	0.0764	0.1529	0.2293	0.1726	0.314	0.487	0.487	0.204	0.281	0.0662	0.1183	0.1845
230	0.2330	0.503	0.736	0.0830	0.1459	0.2289	0.1885	0.298	0.486	0.486	0.194	0.278	0.0722	0.1126	0.1848
248	0.2560	0.475	0.731	0.0895	0.1394	0.2289	0.2042	0.281	0.485	0.485	0.184	0.275	0.0782	0.1069	0.1851
266	0.2785	0.449	0.728	0.0957	0.1332	0.2289	0.2199	0.265	0.485	0.485	0.174	0.271	0.0842	0.1010	0.1852
284	0.3015	0.426	0.728	0.1018	0.1278	0.2296	0.2340	0.250	0.484	0.484	0.164	0.268	0.0899	0.0959	0.1858
302	0.3250	0.403	0.728	0.1077	0.1219	0.2296	—	—	—	—	0.154	0.264	0.0953	0.0905	0.1858
320	—	—	—	0.1135	0.1167	0.2302	—	—	—	—	—	—	0.1003	0.0855	0.1858

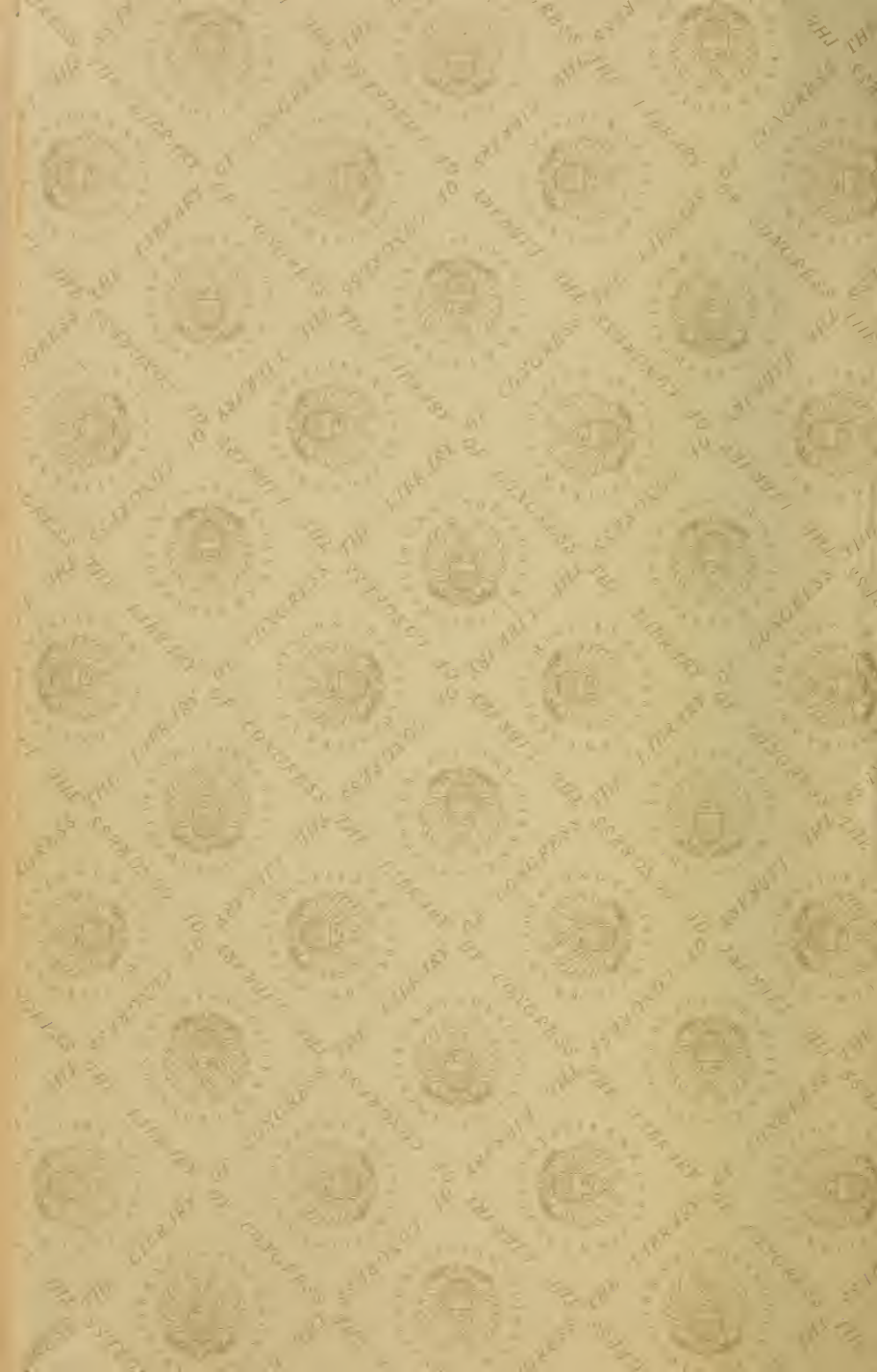
 $n_w$  = entropy of liquid above 32° F. $n_e$  = entropy of vaporization $n_s = n_w + n_e$  = total entropy of dry vapor above 32° F.













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